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Neutrosophic αB*Gα Functions in Neutrosophic Topological Spaces

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Abstract: The notion of a neutrosophic set is generally referred to as the generalization of an intuitionistic fuzzy set. Studying open and closed set variations is crucial in Neutrosophic topology, given the growing significance of Neutrosophic sets in various applications. The origin, nature, and scope of neutrality are explored through the Neutrosophic set. This concept is crucial for research due to its potential applications across various scientific and technological fields. Because the universe inherently contains indeterminacy, the Neutrosophic set provides a valuable framework for study. It is currently being developed to represent data that is uncertain, imprecise, incomplete, or inconsistent. A Neutrosophic set is described using three membership functions: truth, indeterminacy, and falsity. This approach helps to manage uncertainty and leads to more logical outcomes in practical scenarios. Additionally, the Neutrosophic set can identify inconsistencies within data and offer solutions to real-world problems. Neutrosophic functions, based on the Neutrosophic Set Theory, have broad and growing applications due to their ability to model uncertainty, indeterminacy, and inconsistency in data. Here are some of the key areas where neutrosophic functions are applied: Artificial Intelligence & Machine Learning, Data Science and Information Fusion, Decision-Making and Multi-Criteria Decision Analysis (MCDA), Business and Economics, Healthcare and Medical Diagnosis, and Control Systems and Robotics. In 2024, Suthi Keerthana Kumar, Vigneshwaran Mandarasalam, Saied Jafari, and Vidyarani Lakshmanadas described the concepts of Neutrosophic αb*gα-closed sets, Neutrosophic αb*gα-open sets, Neutrosophic αb*gα-border, and Neutrosophic αb*gα-frontier and discussed their properties in Neutrosophic topological spaces. In this research paper, we introduce the concepts of Neutrosophic αb*gα-continuous (Nαb*gα-continuous) maps, Nαb*gα-irresolute maps, Nαb*gα-closed maps, Nαb*gα-open maps, strongly Nαb*gα-continuous maps, perfectly Nαb*gα-continuous maps, contra Nαb*gα-continuous maps, and contra Nαb*gα-irresolute maps in Neutrosophic topological spaces. We investigate and obtain several properties and characterizations concerning these mappings in Neutrosophic topological spaces

Keywords: Neutrosophic αb*gα-continuous map, Neutrosophic αb*gα-irresolute map, Neutrosophic αb*gα-closed map, Neutrosophic αb*gα-open map, Contra neutrosophic αb*gα-irresolute map

Introduction

Many real-life problems in Business, Finance, Medical Sciences, Engineering, and Social Sciences deal with uncertainties. Smarandache studies neutrosophic set as an approach for solving issues that cover unreliable, indeterminacy, and persistent data. Applications of neutrosophic topology depend upon the properties of neutrosophic closed sets, neutrosophic open sets, neutrosophic interior operator, neutrosophic closure operator, and neutrosophic sets. Neutrosophic topological space, neutrosophic αb*gα-open set, neutrosophic αb*gα-closed set, neutrosophic αb*gα-continuous map, neutrosophic αb*gα-irresolute map, neutrosophic αb*gα-closed map, neutrosophic αb*gα-open map, strongly neutrosophic αb*gα-continuous map, perfectly neutrosophic αb*gα-continuous map, contra neutrosophic αb*gα-continuous map, contra neutrosophic αb*gα-irresolute map. We investigate and obtain several properties and characterizations concerning these mappings in Neutrosophic topological spaces

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2-Preliminaries

Definition 2.1. Let X be a non-empty fixed set. A neutrosophic set $(\mathrm{N}_{eu}-\text{set})$ P is an object having the form $P=\{\langle x,\mu_P(x),\sigma_P(x),\gamma_P(x)\rangle : x\in X\}$, where $\mu_P(x)$- represents the degree of membership, $\sigma_P(x)$- represents the degree of indeterminacy, and $\gamma_P(x)$- represents the degree of non-membership.

Definition 2.2. A neutrosophic topology on a non-empty set X is a family T_N of neutrosophic subsets of X satisfying

(i) $0_N, 1_N \in \mathrm{T}_N$.

(ii) $G\bigcap H \in \mathrm{T}_N$ for every $G, H\in \mathrm{T}_N$,

(iii) $\bigcup_{j\in J} G_j \in \mathrm{T}_N$ for every $\{G_j : j\in J\}\subseteq \tau_N$.

Then the pair (X,T_N) is called a neutrosophic topo $\mathrm{N}_{eu}-\text{open}$ logical space $(\mathrm{N}_{eu}-\text{Top}-\text{Space})$. The elements of T_N are called neutrosophic open $(\mathrm{N}_{eu}-\text{open})$ sets in X. A $\mathrm{N}_{eu}-\text{set}$ A is called a neutrosophic closed $(\mathrm{N}_{eu}-\text{closed})$ set if and only if its complement A^C is a $\mathrm{N}_{eu}-\text{open}$ set.

Definition 2.3. Let (X,T_N) be a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ and A be a $\mathrm{N}_{eu}-\text{set}$. Then

(i) The neutrosophic interior of A, denoted by $\mathrm{N}_{eu}-Int(A)$ is the union of all $\mathrm{N}_{eu}-\text{open}$ subsets of X contained in A.

(ii) The neutrosophic closure of A denoted by $\mathrm{N}_{eu}Cl(A)$ is the intersection of all $\mathrm{N}_{eu}-\text{closed}$ sets containing A.

Definition 2.4. Let A be a $\mathrm{N}_{eu}-\text{set}$ in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N). Then the set A is called a

(i) neutrosophic semi-open $\mathrm{N}_{eu}-\text{open}$ $(\mathrm{N}_{eu}\mathbf{s}-\text{open})$ set in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ X if $A\subseteq \mathrm{N}_{eu}Cl[\mathrm{N}_{eu}Int(A)]$.

(ii) neutrosophic semi-closed $(\mathrm{N}_{eu}\mathbf{s}-\text{closed})$ set in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ X if $\mathrm{N}_{eu}Int[\mathrm{N}_{eu}Cl(A)]\subseteq A$.

Definition 2.5. Let A be a $\mathrm{N}_{eu}-\text{set}$ in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N). Then the set A is called a neutrosophic $\alpha-\text{closed}$ (respectively, neutrosophic $\alpha-\text{open}$) set in $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ X if $\mathrm{N}_{eu}Cl[\mathrm{N}_{eu}Int(\mathrm{N}_{eu}Cl(A))]\subseteq A$.

Definition 2.6. Let A be a $\mathrm{N}_{eu}-\text{set}$ in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N). Then the set A is called a neutrosophic generalized closed $(\mathrm{N}_{eu}\mathrm{g}-\text{closed})$ set in $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N) if $\mathrm{N}_{eu}-\mathrm{Cl}(A)\subseteq \mathrm{G}$, whenever $A\subseteq G$ and G is $\mathrm{N}_{eu}-\text{open}$ set in X. $A\subseteq \mathrm{N}_{eu}Int[\mathrm{N}_{eu}Cl(A)]\bigcup \mathrm{N}_{eu}Cl[\mathrm{N}_{eu}Int(A)]$.

Definition 2.7. Let A be a $\mathrm{N}_{eu}-\text{set}$ in a $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N). Then the set A is called a neutrosophic generalized semi-closed $(\mathrm{N}_{eu}\mathrm{gs}-\text{closed})$ set in $\mathrm{N}_{eu}-\text{Top}-\text{Space}$ (X,T_N) if $\mathrm{N}_{eu}-\mathrm{Cl}(A)\subseteq \mathrm{G}$, whenever $A\subseteq G$ and G is $\mathrm{N}_{eu}-\text{open}$ set in X.

Definition 2.8. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a Neutrosophic α-generalized closed$(\mathrm{N}_{eu}\alpha g\text{-}closed)$ set in N_{eu}-Top-Space (X, T_N) if $\mathrm{N}_{eu}\alpha\text{-}Cl(A) \subseteq G$ whenever $A \subseteq G$ and G is $\mathrm{N}_{eu}\text{-}open$ set in X.

Definition 2.9. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a Neutrosophic α^*-open $(\mathrm{N}_{eu}\alpha^*\text{-}open)$ set in N_{eu}-Top-Space (X, T_N) if $A \subseteq \mathrm{N}_{eu}\alpha\text{-}Int\left[\mathrm{N}_{eu}\text{-}Cl\left(\mathrm{N}_{eu}\alpha\text{-}Int(A)\right)\right]$.

Definition 2.10. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a Neutrosophic b-closed $(\mathrm{N}_{eu}b\text{-}closed)$ set in a N_{eu}-Top-Space (X, T_N) if $\left[\mathrm{N}_{eu}\text{-}Cl\left(\mathrm{N}_{eu}\text{-}Int(A)\right)\right] \cup \left[\mathrm{N}_{eu}\text{-}Int\left(\mathrm{N}_{eu}\text{-}Cl(A)\right)\right] \subseteq A$.

Definition 2.11. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called

(i) a Neutrosophic $g\alpha$-open $(\mathrm{N}_{eu}g\alpha\text{-}open)$ set in a N_{eu}-Top-Space (X, T_N) if $V \subseteq \mathrm{N}_{eu}\alpha\text{-}Int(A)$ whenever $V \subseteq A$ and V is a $\mathrm{N}_{eu}\alpha\text{-}closed$ set in (X, T_N).

(ii) a Neutrosophic $g\alpha$-closed $(\mathrm{N}_{eu}g\alpha\text{-}closed)$ set in a N_{eu}-Top-Space (X, T_N) if $\mathrm{N}_{eu}\alpha\text{-}Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is a $\mathrm{N}_{eu}\alpha\text{-}open$ set in (X, T_N).

(iii) a Neutrosophic $^*g\alpha$-open $(\mathrm{N}_{eu}{}^*g\alpha\text{-}open)$ set in a N_{eu}-Top-Space (X, T_N) if $V \subseteq \mathrm{N}_{eu}\text{-}Int(A)$ whenever $V \subseteq A$ and V is a $\mathrm{N}_{eu}g\alpha\text{-}closed$ set in (X, T_N).

(iv) a Neutrosophic $^*g\alpha$-closed $(\mathrm{N}_{eu}{}^*g\alpha\text{-}closed)$ set in a N_{eu}-Top-Space (X, T_N) if $\mathrm{N}_{eu}\text{-}Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is a $\mathrm{N}_{eu}g\alpha\text{-}open$ set in (X, T_N).

Definition 2.12. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a

(i) a Neutrosophic $b^*g\alpha$-open $(\mathrm{N}_{eu}b^*g\alpha\text{-}open)$ set in a N_{eu}-Top-Space (X, T_N) if $V \subseteq \mathrm{N}_{eu}b\text{-}Int(A)$ whenever $V \subseteq A$ and V is a $\mathrm{N}_{eu}{}^*g\alpha\text{-}closed$ set in (X, T_N).

(ii) a Neutrosophic $b^*g\alpha$-closed $(\mathrm{N}_{eu}b^*g\alpha\text{-}closed)$ set in a N_{eu}-Top-Space (X, T_N) if $\mathrm{N}_{eu}b\text{-}Cl(A) \subseteq V$ whenever $A \subseteq V$ and V is a $\mathrm{N}_{eu}{}^*g\alpha\text{-}open$ set in (X, T_N).

Definition 2.13. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a Neutrosophic $\alpha b^*g\alpha\text{-}closed$ $(\mathrm{N}_{eu}\alpha b^*g\alpha\text{-}closed)$ set in N_{eu}-Top-Space (X, T_N) if $\mathrm{N}_{eu}\alpha\text{-}Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is $\mathrm{N}_{eu}b^*g\alpha\text{-}open$ set.

Definition 2.14. Let A be a N_{eu}-set in a N_{eu}-Top-Space (X, T_N). Then the set A is called a Neutrosophic $\alpha b^*g\alpha\text{-}open$ $(\mathrm{N}_{eu}\alpha b^*g\alpha\text{-}open)$ set in N_{eu}-Top-Space (X, T_N) if its complement A^C is a $\mathrm{N}_{eu}\alpha b^*g\alpha\text{-}closed$ set in (X, T_N).

Theorem 2.15. (i) Every $\mathrm{N}_{eu}-closed$ set is $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ set.
(ii) Every $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ set is $\mathrm{N}_{eu}b^*g\alpha-closed$ set.
(iii) Every $\mathrm{N}_{eu}\alpha-closed$ set is $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ set.

Theorem 2.16. Let (X,T_N) be a $\mathrm{N}_{eu}-Top-Space$. Then the union and the intersection of any two $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ sets is a $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ set in $\mathrm{N}_{eu}-Top-Space$ (X,T_N).

Theorem 2.17. Let (X,T_N) be a $\mathrm{N}_{eu}-Top-Space$. Then the intersection and the union of any two $\mathrm{N}_{eu}\alpha b^*g\alpha-open$ sets is a $\mathrm{N}_{eu}\alpha b^*g\alpha-open$ set in $\mathrm{N}_{eu}-Top-Space$ (X,T_N).

Definition 2.18. A $\mathrm{N}_{eu}-set\ A$ in a $\mathrm{N}_{eu}-Top-Space.$ (X,T_N) is called aneutrosophic $\alpha b^*g\alpha-intterior$ of A $(\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A))$ and neutrosophic $\alpha b^*g\alpha-closure$ of A $(\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A))$ are respectively defined by
$\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)=\bigcup\{G:G\in \mathrm{N}_{eu}\alpha b^*g\alpha-Int(X,T_{Neu})\ and\ G\subseteq A\}$ and
$\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)=\bigcup\{G:G\in \mathrm{N}_{eu}\alpha b^*g\alpha-Int(X,T_{Neu})\ and\ G\subseteq A\}$.

Remark 2.19. Let A be a subset of a $\mathrm{N}_{eu}-Top-Space.$ (X,T_N). Then $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)$ $(\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A))$ is $\mathrm{N}_{eu}\alpha b^*g\alpha-open$ $(\mathrm{N}_{eu}\alpha b^*g\alpha-closed)$ set in (X,T_N). The complement of $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)$ is $\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)$.

Definition 2.20. Let A be a $\mathrm{N}_{eu}-subset$ of a $\mathrm{N}_{eu}-Top-Space.$ (X,T_N). Then the neutrosophic $\alpha\mathrm{b}^*g\alpha-\mathrm{frontier}$ of a $\mathrm{N}_{eu}-subset\ A$ of X is denoted by $\mathrm{N}_{eu}\alpha b^*g\alpha-Fr(A)$ and is defined by $\mathrm{N}_{eu}\alpha b^*g\alpha-Fr(A)=\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)\right]\bigcap\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A^C)\right]$.

Theorem 2.21. For $\mathrm{N}_{eu}-sets\ A$ and B in a $\mathrm{N}_{eu}-Top-Space.$ (X,T_N), the following statements are true:
(i) $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\subseteq A\subseteq \mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)$.
(ii) A is $\mathrm{N}_{eu}\alpha b^*g\alpha-open$ set in X if and only if $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)=A$.
(iii) A is $\mathrm{N}_{eu}\alpha b^*g\alpha-closed$ set in X if and only if $\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)=A$.
(iv) $\mathrm{N}_{eu}\alpha b^*g\alpha-Int\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\right]=\mathrm{N}_{eu}gs\alpha^*-Int(A)$.
(v) $\mathrm{N}_{eu}\alpha b^*g\alpha-Cl\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)\right]=\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)$.
(vi) If $A\subseteq B$, then $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\subseteq \mathrm{N}_{eu}\alpha b^*g\alpha-Int(B)$.
(vii) $\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\right]^C=\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A^C)$.
(viii) $\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)\right]^C=\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A^C)$.
(ix) $\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A\bigcap B)=\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\right]\bigcap\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(B)\right]$.
(x) $\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A\bigcup B)=\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(A)\right]\bigcup\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Cl(B)\right]$.
(xi) $\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(A)\right]\bigcup\left[\mathrm{N}_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq \mathrm{N}_{eu}\alpha b^*g\alpha-Int(A\bigcup B)$.

(xii) $N_{eu}\alpha b^*g\alpha-Cl(A\cap B)\subseteq[N_{eu}\alpha b^*g\alpha-Cl(A)]\cap[N_{eu}\alpha b^*g\alpha-Cl(B)]$.

Definition 2.22. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a mapping. Then f is called a neutrosophic continuous ($N_{eu}-continuous$) mapping if $f^{-1}(V)$ is a $N_{eu}-open$ set in X for every $N_{eu}-open$ set V in Y.

Theorem 2.23. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a mapping. Then f is called a $N_{eu}-continuous$ mapping if $f^{-1}(V)$ is a $N_{eu}-closed$ set in X for every $N_{eu}-closed$ set V in Y.

3 Neutrosophic $\alpha b^*g\alpha-$Continuous Mappings

In this section, we introduce the concepts of neutrosophic $\alpha b^*g\alpha-$continuous ($N_{eu}\alpha b^*g\alpha-continuous$) mappings in $N_{eu}-Top-Spaces$. Also, we study some of the main results regarding $N_{eu}\alpha b^*g\alpha-continuous$ depending on $N_{eu}\alpha b^*g\alpha-open$ sets.

Definition 3.1. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a mapping. Then f is called a $N_{eu}\alpha b^*g\alpha-continuous$ mapping if $f^{-1}(V)$ is a $N_{eu}\alpha b^*g\alpha-open$ set in X for every $N_{eu}-open$ set V in Y.

Theorem 3.2. Every $N_{eu}-continuous$ mapping is $N_{eu}\alpha b^*g\alpha-continuous$ mapping.

Proof. Let $f:(X,T_N)\to(Y,\sigma_N)$ be $N_{eu}-continuous$ mapping. Let V be a $N_{eu}-open$ set in (Y,σ_N). Then $f^{-1}(V)$ is $N_{eu}-open$ set in (X,T_N). Since every $N_{eu}-open$ set is $N_{eu}\alpha b^*g\alpha-open$. $f^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-open$ set in (X,T_N). Hence f is $N_{eu}\alpha b^*g\alpha-continuous$ mapping.

Theorem 3.3. Let (X,T_N), (Y,σ_N) and (Z,η_N) be $N_{eu}-Top-Spaces$. If $f:(X,T_N)\to(Y,\sigma_N)$ is a $N_{eu}\alpha b^*g\alpha-continuous$ mapping and $g:(Y,\sigma_N)\to(Z,\eta_N)$ is $N_{eu}-continuous$ mapping, then $gof:(X,T_N)\to(Z,\eta_N)$ is a $N_{eu}\alpha b^*g\alpha-continuous$ mapping.

Proof. Let G be a $N_{eu}-open$ set in Z. Since $g:(Y,\sigma_N)\to(Z,\eta_N)$ is $N_{eu}-continuous$, $f^{-1}(G)$ is $N_{eu}-open$ in Y. Since f is a $N_{eu}\alpha b^*g\alpha-continuous$ mapping, $f^{-1}[f^{-1}(G)]$ is $N_{eu}\alpha b^*g\alpha-open$ in X. But $f^{-1}[g^{-1}(G)]=(gof)^{-1}(G)$.
Then $(gof)^{-1}(G)$ is $N_{eu}\alpha b^*g\alpha-open$ set in X. Hence, gof is a $N_{eu}\alpha b^*g\alpha-continuous$ mapping.

Theorem 3.4. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$. Then prove that $f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}\alpha b^*g\alpha-continuous$ if and only if $f^{-1}(B)$ is $N_{eu}\alpha b^*g\alpha-closed$ set in X for every $N_{eu}-closed$ set B in Y.

Proof. Let B be a N_{eu} *-closed* set in Y. Then B^C is N_{eu} *-open* set in Y. Since f is $N_{eu}\alpha b^* g\alpha$ *-continuous*. Therefore $f^{-1}\left(B^C\right)$ is a $N_{eu}\alpha b^* g\alpha$ *-open* set in X. Since $f^{-1}\left(B^C\right)=\left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is $N_{eu}\alpha b^* g\alpha$ *-closed* set in X.

Conversely, Let B be a N_{eu} *-open* set in Y. Then B^C is N_{eu} *-closed* set in Y. By assumption $f^{-1}\left(B^C\right)$ is $N_{eu}\alpha b^* g\alpha$ *-closed* set in X. Since $f^{-1}\left(B^C\right)=\left[f^{-1}(B)\right]^C$, $f^{-1}(B)$ is $N_{eu}\alpha b^* g\alpha$ *-open* set in X. Hence f is $N_{eu}\alpha b^* g\alpha$ *-continuous*.

Theorem 3.5. Let (X,T_N) and (Y,σ_N) be two N_{eu} *-Top-Spaces* and $f: X \rightarrow Y$ be a mapping. Then f is a $N_{eu}\alpha b^* g\alpha$ *-continuous* mapping if and only if $f\left(N_{eu}\alpha b^* g\alpha\text{-}Cl(A)\right) \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f(A)\right)$ for every N_{eu} *-set* A in X.

Proof. Let A be a N_{eu} *-set* in X and f be a $N_{eu}\alpha b^* g\alpha$ *-continuous* mapping. Then evidently $f(A) \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f(A)\right]$. Now, $A \subseteq f^{=1}\left[f(A)\right] \subseteq f^{-1}\left[N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f(A)\right)\right]$ and $N_{eu}\alpha b^* g\alpha\text{-}Cl(A) \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}\left(N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f(A)\right)\right)\right]$. Since f is a $N_{eu}\alpha b^* g\alpha$ -continuous mapping and $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f(A)\right]$ is a $N_{eu}\alpha b^* g\alpha$ *-closed* set. Thus $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}\left(N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f(A)\right)\right)\right] = f^{-1}\left[N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f(A)\right)\right]$. Hence, $f\left[N_{eu}\alpha b^* g\alpha\text{-}Cl(A)\right] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f(A)\right]$.

Conversely, let $f\left[N_{eu}\alpha b^* g\alpha\text{-}Cl(A)\right] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f(A)\right]$, for each N_{eu} *-set* A in X. Let F be a N_{eu} *-closed* set in Y. Then $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f\left(f^{-1}(F)\right)\right] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl(F) = F$. By assumption, $f\left[N_{eu}\alpha b^* g\alpha\text{-}Cl\left(f^{-1}(F)\right)\right] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f\left(f^{-1}(F)\right)\right] \subseteq F$ and hence $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}(F)\right] \subseteq f^{-1}(F)$. Since $f^{-1}(F) \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}(F)\right]$, $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}(F)\right] = f^{-1}(F)$. This implies that $f^{-1}(F)$ is a $N_{eu}\alpha b^* g\alpha$ *-closed* set in X. Thus by Theorem 3.4, f is a $N_{eu}\alpha b^* g\alpha$ -continuous mapping.

Theorem 3.6. Let (X,T_N) and (Y,σ_N) be two N_{eu} *-Top-Spaces* and $f: X \rightarrow Y$ be a mapping. Then f is a $N_{eu}\alpha b^* g\alpha$ *-continuous* mapping if and only if $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}(B)\right] \subseteq f^{-1}\left[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)\right]$ for every N_{eu} *-set* B in Y.

Proof. Let B be any N_{eu} *-set* in Y and f be a $N_{eu}\alpha b^* g\alpha$ *-continuous* mapping. Clearly $f^{-1}(B) \subseteq f^{-1}\left[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)\right]$. Then, $N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}(B)\right] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl\left[f^{-1}\left(N_{eu}\alpha b^* g\alpha\text{-}Cl(B)\right)\right]$. Since $N_{eu}\alpha b^* g\alpha\text{-}Cl(B)$ is $N_{eu}\alpha b^* g\alpha$ -closed set in Y. So by Theorem 3.4, $f^{-1}\left[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)\right]$ is a $N_{eu}\alpha b^* g\alpha$ *-cloed* set in X. Thus,

$N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(B)\right]\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Cl(B)\right)\right]=$
$f^{-1}\left[N_{eu}\alpha b^*g\alpha-Cl(B)\right].$

Conversely, $N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^*g\alpha-Cl(B)\right]$ for all $N_{eu}-sets$ $N_{eu}-set$ B in Y. Let F be a $N_{eu}-closed$ set in Y. Since every $N_{eu}-closed$ set is $N_{eu}\alpha b^*g\alpha-closed$ set, $N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(F)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^*g\alpha-Cl(F)\right]=f^{-1}(F)$. This implies that $f^{-1}(F)$ is a $N_{eu}\alpha b^*g\alpha-closed$ set in X. Thus by Theorem 3.4, f is a $N_{eu}\alpha b^*g\alpha-continuous$ mapping.

Theorem 3.7. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$ and $f:X\to Y$ be a bijective mapping. Then f is $N_{eu}\alpha b^*g\alpha$-continuous if and only if $N_{eu}\alpha b^*g\alpha-Int\left[f(A)\right]\subseteq f\left[N_{eu}\alpha b^*g\alpha-Int(A)\right]$ for every $N_{eu}-set$ A in X.

Proof. Let A be any $N_{eu}-set$ in X and f be a bijective and $N_{eu}\alpha b^*g\alpha$-continuous mapping. Let $f(A)=B$. Clearly $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq f^{-1}(B)$. Since f is an injective mapping, $f^{-1}(B)=A$, so that $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq A$. Therefore, $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Int(B)\right)\right]\subseteq N_{eu}\alpha b^*g\alpha-Int(A)$. Since f is $N_{eu}\alpha b^*g\alpha$-continuous, $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]$ is $N_{eu}\alpha b^*g\alpha$-open set in X and $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq$ $N_{eu}\alpha b^*g\alpha-Int(A)$, $f\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Int(B)\right)\right]\subseteq$ $f\left[N_{eu}\alpha b^*g\alpha-Int(A)\right]$. Therefore $N_{eu}\alpha b^*g\alpha-Int\left[f(A)\right]\subseteq$ $f\left[N_{eu}gs\alpha^*-Int(A)\right]$.

Conversely, $N_{eu}\alpha b^*g\alpha-Int\left[f(A)\right]\subseteq f\left[N_{eu}\alpha b^*g\alpha-Int(A)\right]$ for every $N_{eu}-set$ A in X. Let V be a $N_{eu}-open$ set in Y. Then V is $N_{eu}\alpha b^*g\alpha$-open set in Y. Since f is surjective and so $V=N_{eu}\alpha b^*g\alpha-Int(V)=N_{eu}\alpha b^*g\alpha-Int\left[f\left(f^{-1}(V)\right)\right]\subseteq f\left[N_{eu}\alpha b^*g\alpha-Int\left(f^{-1}(V)\right)\right]$. It follows that $f^{-1}(V)\subseteq$ $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(V)\right]$. Therefore $f^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha$-open set in X. Hence f is a $N_{eu}\alpha b^*g\alpha$-continuous mapping.

Theorem 3.8. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$ and $f:X\to Y$ be a mapping. Then f is a $N_{eu}\alpha b^*g\alpha$-continuous mapping if and only if $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(B)\right]$ for every $N_{eu}-set$ B in Y.

Proof. Let B be any $N_{eu}-set$ in Y and f be a $N_{eu}\alpha b^*g\alpha$-continuous mapping. Clearly $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq f^{-1}(B)$ implies $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Int(B)\right)\right]\subseteq$ $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(B)\right]$. Since $N_{eu}\alpha b^*g\alpha-Int(B)$ is $N_{eu}\alpha b^*g\alpha$-open set in Y and f is $N_{eu}\alpha b^*g\alpha$-continuous, $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]$ is $N_{eu}\alpha b^*g\alpha$-open set in X. Therefore $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Int(B)\right)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]\subseteq$
$N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(B)\right].$

Conversely, $f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Int(B)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Int\left[f^{-1}(B)\right]$ for every $N_{eu}-set$ B in Y. Let G be any $N_{eu}-open$ set in Y. Then $f^{-1}(G)=f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Int(G)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Int\left[f^{-1}(G)\right]$ and therefore $f^{-1}(G)=N_{eu}\alpha b^{*}g\alpha-Int\left[f^{-1}(G)\right]$. This implies that $f^{-1}(G)$ is $N_{eu}\alpha b^{*}g\alpha-$open set in X. Hence f is a $N_{eu}\alpha b^{*}g\alpha-$continuous mapping.

Theorem 3.9. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$ and $f:X\to Y$ be a bijective mapping. Then f is a $N_{eu}\alpha b^{*}g\alpha-$continuous mapping if and only if $f\left[N_{eu}\alpha b^{*}g\alpha-Fr(A)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Fr\left[f(A)\right]$ for every $N_{eu}-set$ A in X.

Proof. Let f be a bijective and $N_{eu}\alpha b^{*}g\alpha-$continuous mapping. Let A be a $N_{eu}-set$ in X. By definition, $N_{eu}\alpha b^{*}g\alpha-Fr(A)=N_{eu}\alpha b^{*}g\alpha-Cl(A)\cap N_{eu}\alpha b^{*}g\alpha-Cl(A^{C})$. By Theorem 3.7, $N_{eu}\alpha b^{*}g\alpha-Int\left[f(A)\right]\subseteq f\left[N_{eu}\alpha b^{*}g\alpha-Int(A)\right]$ and from Theorem 3.5, $f\left[N_{eu}\alpha b^{*}g\alpha-Cl(A)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Cl\left[f(A)\right]$,

$f\left[N_{eu}\alpha b^{*}g\alpha-Fr(A)\right]=f\left[N_{eu}\alpha b^{*}g\alpha-Cl(A)\right]\cap f\left[N_{eu}\alpha b^{*}g\alpha-Cl\left(A^{C}\right)\right]\subseteq$

$N_{eu}\alpha b^{*}g\alpha-Cl\left[f(A)\right]\cap N_{eu}\alpha b^{*}g\alpha-Cl\left[f(A)\right]^{C}=N_{eu}\alpha b^{*}g\alpha-Fr\left[f(A)\right]$.

Conversely, $f\left[N_{eu}\alpha b^{*}g\alpha-Fr(A)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Fr\left[f(A)\right]$ for every $N_{eu}-set$ A in X. Then $f\left[N_{eu}\alpha b^{*}g\alpha-Cl(A)\right]=f\left[N_{eu}\alpha b^{*}g\alpha-Int(A)\right]\cup f\left[N_{eu}\alpha b^{*}g\alpha-Fr(A)\right]\subseteq$ $f(A)\cup N_{eu}\alpha b^{*}g\alpha-Fr\left[f(A)\right]\subseteq N_{eu}\alpha b^{*}g\alpha-Cl\left[f(A)\right]$. By Theorem 3.5, f is a $N_{eu}\alpha b^{*}g\alpha-$continuous mapping.

Theorem 3.10. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$ and $f:X\to Y$ be a bijective mapping. Then f is a $N_{eu}\alpha b^{*}g\alpha-$continuous mapping if and only if $N_{eu}\alpha b^{*}g\alpha-Fr\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Fr(B)\right]$ for every $N_{eu}-set$ B in Y.

Proof. Let f be a bijective and $N_{eu}\alpha b^{*}g\alpha-$continuous mapping. Let B be a $N_{eu}-set$ in Y. By Theorem 3.6, $N_{eu}\alpha b^{*}g\alpha-Cl\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Cl(B)\right]$. Therefore we obtain

$f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Fr(B)\right]=f^{-1}\left[\left(N_{eu}\alpha b^{*}g\alpha-Cl(B)\right)\cap N_{eu}\alpha b^{*}g\alpha-Cl\left(B^{C}\right)\right]=$

$f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Cl(B)\right]\cap f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Cl\left(B^{C}\right)\right]\supseteq$

$N_{eu}\alpha b^{*}g\alpha-Cl\left[f^{-1}(B)\right]\cap N_{eu}\alpha b^{*}g\alpha-Cl\left[f^{-1}\left(B^{C}\right)\right]=$

$N_{eu}\alpha b^{*}g\alpha-Cl\left[f^{-1}(B)\right]\cap N_{eu}\alpha b^{*}g\alpha-Cl\left[\left(f^{-1}(B)\right)^{C}\right]=N_{eu}\alpha b^{*}g\alpha-Fr\left[f^{-1}(B)\right]$.

Therefore $N_{eu}\alpha b^{*}g\alpha-Fr\left[f^{-1}(B)\right]\subseteq f^{-1}\left[N_{eu}\alpha b^{*}g\alpha-Fr(B)\right]$.

Conversely since $N_{eu}\alpha b^* g\alpha - Fr\left[f^{-1}(B)\right] \subseteq f^{-1}\left[N_{eu}\alpha b^* g\alpha - Fr(B)\right]$ for every $N_{eu}-set$ B in Y. This implies that $N_{eu}\alpha b^* g\alpha - Cl\left[f^{-1}(B)\right] \subseteq f^{-1}\left[N_{eu}\alpha b^* g\alpha - Cl(B)\right]$. By Theorem 3.6, f is a $N_{eu}\alpha b^* g\alpha -$continuous mapping.

Definition 3.11. Let $x_{(r,t,s)}$ be a $N_{eu}-point$ of a $N_{eu}-Top-Space\,(X,T_N)$. A $N_{eu}-set$ A of X is called neutrosophic neighbourhood $(N_{eu}-neighbourhood)$ of $x_{(r,t,s)}$ if there exists a $N_{eu}-open$ set B such that $x_{(r,t,s)} \in B \subseteq A$.

Theorem 3.12. Let f be a mapping from a $N_{eu}-Top-Space$ (X,T_N) to a $N_{eu}-Top-Space$ (Y,σ_N). Then the following assertions are equivalent.

(i) f is $N_{eu}\alpha b^* g\alpha -$continuous.

(ii) For each $N_{eu}-point$ $x_{(r,t,s)} \in X$ and every $N_{eu}-neighbourhood$ A of $f\left(x_{(r,t,s)}\right)$, there exists a $N_{eu}\alpha b^* g\alpha -$open set B such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$.

(iii) For each $N_{eu}-point$ $x_{(r,t,s)} \in X$ and every $N_{eu}-neighbourhood$ A of $f\left(x_{(r,t,s)}\right)$, there exists a $N_{eu}\alpha b^* g\alpha -$open set B in X such that $x_{(r,t,s)} \in B$ and $f(B) \subseteq A$.

Proof. $(\text{i}) \Rightarrow (\text{ii})$: Let $x_{(r,t,s)} \in X$ be a $N_{eu}-point$ in X and let A be a $N_{eu}-neighbourhood$ of $f\left(x_{(r,t,s)}\right)$. Then there exists a $N_{eu}-open$ set B in Y such that $f\left(x_{(r,t,s)}\right) \in B \subseteq A$. Since f is $N_{eu}\alpha b^* g\alpha -$continuous, we know that $f^{-1}(B)$ is a $N_{eu}\alpha b^* g\alpha -$open set in X and $x_{(r,t,s)} \in f^{-1}\left(f_{(r,t,s)}\right) \subseteq f^{-1}(B) \subseteq f^{-1}(A)$. This implies (ii) is true.

$(\text{ii}) \Rightarrow (\text{iii})$: Let $x_{(r,t,s)}$ be a $N_{eu}-point$ in X and let A be a $N_{eu}-neighbourhood$ of $f\left(x_{(r,t,s)}\right)$. The condition (ii) implies that there exists a $N_{eu}\alpha b^* g\alpha -$open set B in X such that $x_{(r,t,s)} \in B \subseteq f^{-1}(A)$. Thus $x_{(r,t,s)} \in B$ and $f(B) \subseteq f\left[f^{-1}(A)\right] \subseteq A$. Hence (iii) is true.

$(\text{iii}) \Rightarrow (\text{i})$: Let B be a $N_{eu}-open$ set in Y and let $x_{(r,t,s)} \in f^{-1}(B)$. Since B is $N_{eu}-open$ set, $f\left(x_{(r,t,s)}\right) \in B$, and so B is $N_{eu}-neighbourhood$ of $f\left(x_{(r,t,s)}\right)$. It follows from (iii) that there exists a $N_{eu}\alpha b^* g\alpha -$open set A in X such that $x_{(r,t,s)} \in A$ and $f(A) \subseteq B$ so that $x_{(r,t,s)} \in A \subseteq f^{-1}\left[f(A)\right] \subseteq f^{-1}(B)$. This implies by definition that $f^{-1}(B)$ is a $N_{eu}\alpha b^* g\alpha -$open set in X. Therefore, f is a $N_{eu}\alpha b^* g\alpha -$continuous mapping.

4 Neutrosophic $\alpha b^* g\alpha$ –Irresolute Mappings

In this section, we introduce the concept of neutrosophic $N_{eu}\alpha b^* g\alpha -$irresolute $(N_{eu}\alpha b^* g\alpha -$irresolute$)$ mappings in $N_{eu}-Top-Spaces$. Also, we discuss the relationship of $N_{eu}\alpha b^* g\alpha -$irresolute with $N_{eu}\alpha b^* g\alpha -$continuous mappings.

Definition 4.1. Let (X, T_N) and (Y, σ_N) be two N_{eu} *-Top -Spaces*. A mapping $f : X \to Y$ is called $N_{eu}\alpha b^* g\alpha$-irresolute if the inverse image of every $N_{eu}\alpha b^* g\alpha$-open set in Y is $N_{eu}\alpha b^* g\alpha$-open in X.

Theorem 4.2. Let (X, T_N) and (Y, σ_N) be two N_{eu} *-Top -Spaces*. A mapping $f : X \to Y$ is called $N_{eu}\alpha b^* g\alpha$-irresolute if the inverse image of every $N_{eu}\alpha b^* g\alpha$-closed set in Y is $N_{eu}\alpha b^* g\alpha$-closed in X.

Proof. Let A be any $N_{eu}\alpha b^* g\alpha$-closed set in Y. Then A^C is $N_{eu}\alpha b^* g\alpha$-open set in Y. Since f is $N_{eu}\alpha b^* g\alpha$-irresolute, $f^{-1}(A^C)$ is $N_{eu}\alpha b^* g\alpha$-open set in X and $f^{-1}(A^C) = [f^{-1}(A)]^C$ which implies that $f^{-1}(A)$ is $N_{eu}\alpha b^* g\alpha$-closed set in X.
Conversely, Let B be any $N_{eu}\alpha b^* g\alpha$--open set in Y. Then B^C is $N_{eu}\alpha b^* g\alpha$-closed set in Y. Thus $f^{-1}(B^C)$ is $N_{eu}\alpha b^* g\alpha$-closed set in X and $f^{-1}(B^C) = [f^{-1}(B)]^C$ which implies that $f^{-1}(B)$ is $N_{eu}\alpha b^* g\alpha$-open set in X. Hence $f : X \to Y$ is $N_{eu}\alpha b^* g\alpha$-irresolute.

Theorem 4.3. Every $N_{eu}\alpha b^* g\alpha$-irresolute mapping is $N_{eu}\alpha b^* g\alpha$-continuous.

Proof. Let V be a N_{eu} *-open* set in Y. Since every N_{eu} *-open* set is $N_{eu}\alpha b^* g\alpha$-open, V is $N_{eu}\alpha b^* g\alpha$-open. Since f is $N_{eu}\alpha b^* g\alpha$-irresolute, $f^{-1}(V)$ is $N_{eu}\alpha b^* g\alpha$-open set in X. Therefore f is $N_{eu}\alpha b^* g\alpha$-continuous.

Theorem 4.4. Let $f : (X, T_N) \to (Y, \sigma_N)$ be a mapping. Then the following assertions are equivalent:
(i) f is $N_{eu}\alpha b^* g\alpha$-irresolute.
(ii) $N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(B)] \subseteq f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)]$ for every N_{eu} *-set* B of Y.
(iii) $f[N_{eu}\alpha b^* g\alpha\text{-}Cl(A)] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl[f(A)]$ for every N_{eu} *-set* A of X.
(iv) $f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Int(B)] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Int[f^{-1}(B)]$ for every N_{eu} *-set* B of Y.

Proof. (i)$\Rightarrow$(ii): Let B be any N_{eu} *-set* in Y. Then $N_{eu}\alpha b^* g\alpha\text{-}Cl(B)$ is $N_{eu}\alpha b^* g\alpha$-closed set in Y. Since f is $N_{eu}\alpha b^* g\alpha$-irresolute, $f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)]$ is $N_{eu}\alpha b^* g\alpha$-closed set in X. Then $N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(N_{eu}\alpha b^* g\alpha\text{-}Cl(B))] = f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)]$. Clearly it follows that $N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(B)] \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(N_{eu}\alpha b^* g\alpha\text{-}Cl(B))] = f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(B)]$. This proves (ii).
(ii)$\Rightarrow$(iii): Let A be any N_{eu} *-set* in X. Then $f(A) \subseteq Y$. By (ii), $N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(f(A))] \subseteq f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(f(A))]$...(*). Now we observe that $A \subseteq f^{-1}(f(A))$ implies that $N_{eu}\alpha b^* g\alpha\text{-}Cl(A) \subseteq N_{eu}\alpha b^* g\alpha\text{-}Cl[f^{-1}(f(A))]$...(**). Then (*) and (**) implies that $N_{eu}\alpha b^* g\alpha\text{-}Cl(A) \subseteq f^{-1}[N_{eu}\alpha b^* g\alpha\text{-}Cl(f(A))]$ which implies that

$f\left[N_{eu}\alpha b^*g\alpha-Cl(A)\right]\subseteq f\left(f^{-1}\left[N_{eu}\alpha b^*g\alpha-Cl\left(f(A)\right)\right]\right)\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]$. Thus, $f\left[N_{eu}\alpha b^*g\alpha-Cl(A)\right]\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]$. Hence, $(\text{ii})\Rightarrow(\text{iii})$ is proved.

$(\text{iii})\Rightarrow(\text{i})$: Let F be any $N_{eu}\alpha b^*g\alpha-$closed set in Y. Then $f^{-1}(F)=f^{-1}\left[N_{eu}\alpha b^*g\alpha-Cl(F)\right]$. By (iii), $f\left[N_{eu}\alpha b^*g\alpha-Cl\left(f^{-1}(F)\right)\right]\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f\left(f^{-1}(F)\right)\right]\subseteq N_{eu}\alpha b^*g\alpha-Cl(F)=F$. Then That implies $N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(F)\right]\subseteq f^{-1}(F)$. But $f^{-1}(F)\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(F)\right]$, $N_{eu}\alpha b^*g\alpha-Cl\left[f^{-1}(F)\right]=f^{-1}(F)$ and so $f^{-1}(F)$ is $N_{eu}\alpha b^*g\alpha-$closed set in X. Therefore f is $N_{eu}\alpha b^*g\alpha-$irresolute.

$(\text{i})\Rightarrow(\text{iv})$: Let B be any $N_{eu}-set$ in Y. We know that $N_{eu}\alpha b^*g\alpha-Int(B)$ is $N_{eu}\alpha b^*g\alpha-$open set in Y. Since f is $N_{eu}\alpha b^*g\alpha-$irresolute, $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]$ is $N_{eu}\alpha b^*g\alpha-$open set in X. Then $f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(B)\right]=N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}\left(N_{eu}\alpha b^*g\alpha-Int(B)\right)\right]\subseteq N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(B)\right]$.

$(\text{iv})\Rightarrow(\text{i})$: Let V be any $N_{eu}\alpha b^*g\alpha-$open set in Y. Then by (iv), $f^{-1}(V)=f^{-1}\left[N_{eu}\alpha b^*g\alpha-Int(V)\right]\subseteq N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(V)\right]$. But, we have $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(V)\right]\subseteq f^{-1}(V)$, $N_{eu}\alpha b^*g\alpha-Int\left[f^{-1}(V)\right]=f^{-1}(V)$ and hence $f^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-$open. Thus f is $N_{eu}\alpha b^*g\alpha-$irresolute.

Theorem 4.5. If $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ are $N_{eu}\alpha b^*g\alpha-$irresolute, then their composition $gof:(X,T_N)\to(Z,\eta_N)$ is also $N_{eu}\alpha b^*g\alpha-$irresolute.

Proof. Let V be a $N_{eu}\alpha b^*g\alpha-$open set in Z. Since g is a $N_{eu}\alpha b^*g\alpha-$irresolute mapping, $g^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-$open in Y. Since f is a $N_{eu}\alpha b^*g\alpha-$irresolute mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-$open in X. Therefore gof is $N_{eu}\alpha b^*g\alpha-$irresolute.

Theorem 4.6. If $f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}\alpha b^*g\alpha-$irresolute and $g:(Y,\sigma_N)\to(Z,\eta_N)$ is $N_{eu}\alpha b^*g\alpha-$continuous, then their composition $gof:(X,T_N)\to(Z,\eta_N)$ is also $N_{eu}\alpha b^*g\alpha-$continuous.

Proof. Let V be a $N_{eu}-open$ set in Z. Since g is a $N_{eu}\alpha b^*g\alpha-$continuous mapping, $g^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-$open set in Y. Since f is a $N_{eu}\alpha b^*g\alpha-$irresolute mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha-$open in X. Therefore gof is $N_{eu}\alpha b^*g\alpha-$conrinuous.

5 Neutrosophic $\alpha b^*g\alpha-$Closed Mappings and Neutrosophic $\alpha b^*g\alpha-$Open Mappings

In this section, we introduce neutrosophic $\alpha b^*g\alpha-$ closed $\left(N_{eu}\alpha b^*g\alpha-\text{closed}\right)$ mappings and neutrosophic $\alpha b^*g\alpha-$ open $\left(N_{eu}\alpha b^*g\alpha-\text{open}\right)$ mappings in $N_{eu}-Top-Spaces$ and obtain certain characterizations of these classes of mappings.

Definition 5.1. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$. A function $f:(X,T_N)\to(Y,\sigma_N)$ is said to be $N_{eu}\alpha b^*g\alpha-\text{closed}$ if the image of each $N_{eu}-\text{closed}$ set in X is $N_{eu}\alpha b^*g\alpha-\text{closed}$ in Y.

Definition 5.2. Let (X,T_N) and (Y,σ_N) be two $N_{eu}-Top-Spaces$. A function $f:(X,T_N)\to(Y,\sigma_N)$ is said to be $N_{eu}\alpha b^*g\alpha-\text{open}$ if the image of each $N_{eu}-\text{open}$ set in X is $N_{eu}\alpha b^*g\alpha-\text{closed}$ in Y.

Theorem 5.3. A function $f:(X,T_N)\to(Y,\sigma_N)$ is said to be $N_{eu}\alpha b^*g\alpha-\text{closed}$ if and only if $N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]\subseteq f\left[N_{eu}Cl(A)\right]$ for every $N_{eu}-\text{set}$ A of X.

Proof. Suppose $f:(X,T_N)\to(Y,\sigma_N)$ is a $N_{eu}\alpha b^*g\alpha-\text{closed}$ function and A is any $N_{eu}-\text{set}$ in X. Then $N_{eu}Cl(A)$ is a $N_{eu}\alpha b^*g\alpha-\text{closed}$ set in X. Since f is $N_{eu}\alpha b^*g\alpha-\text{closed},$ $f\left[N_{eu}Cl(A)\right]$ is a $N_{eu}\alpha b^*g\alpha-\text{closed}$ set in Y. Thus $N_{eu}\alpha b^*g\alpha-Cl\left[f\left(N_{eu}Cl(A)\right)\right]=f\left[N_{eu}Cl(A)\right]$. Therefore $N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]\subseteq$ $N_{eu}\alpha b^*g\alpha-Cl\left[f\left(N_{eu}Cl(A)\right)\right]=f\left(N_{eu}Cl(A)\right)$. Hence $N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]\subseteq$ $f\left(N_{eu}Cl(A)\right)$.

Conversely, let A be a $N_{eu}-\text{closed}$ set in X. Then $N_{eu}Cl(A)=A$ and so $f(A)=f\left[N_{eu}Cl(A)\right]$. By our assumption $N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]\subseteq f(A)$. But $f(A)\subseteq N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]$. Hence $N_{eu}\alpha b^*g\alpha-Cl\left[f(A)\right]=f(A)$ and therefore $f(A)$ is $N_{eu}\alpha b^*g\alpha-\text{closed}$ set in Y. Thus f is a $N_{eu}\alpha b^*g\alpha-\text{closed}$ mapping.

Theorem 5.4.. A mapping $f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}\alpha b^*g\alpha-\text{closed}$ if and only if for each $N_{eu}-\text{set}$ W of Y and for each $N_{eu}-\text{open}$ set U of X containing $f^{-1}(W)$ there exists a $N_{eu}\alpha b^*g\alpha-\text{open}$ set V of Y such that $W\subseteq V$ and $f^{-1}(V)\subseteq U$.

Proof. Suppose f is a $N_{eu}\alpha b^*g\alpha-\text{closed}$ mapping. Let W be any $N_{eu}-\text{set}$ in Y and U be a $N_{eu}\alpha b^*g\alpha-\text{open}$ set of X such that $f^{-1}(W)\subseteq U$. Then $V=\left[f\left(U^c\right)\right]^c$ is $N_{eu}\alpha b^*g\alpha-\text{open}$ set containing W such that $f^{-1}(V)\subseteq U$. Conversely, let W be a $N_{eu}-\text{closed}$ set of X. Then $f^{-1}\left[\left(f(W)\right)^c\right]\subseteq W^c$ and W^c is $N_{eu}-\text{open}$ in X. By assumption, there exists a $N_{eu}\alpha b^*g\alpha-\text{open}$ set V of Y such that $\left[f(W)\right]^c\subseteq V$ and $f^{-1}(V)\subseteq W^c$ and so $W\subseteq\left[f^{-1}(V)\right]^c$. Hence

$V^c \subseteq f(W) \subseteq f\left[\left(f^{-1}(V)\right)^c\right] \subseteq V^c$, which implies $f(W) = V^c$. Since V^c is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed, $f(W)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed and f is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping.

Theorem 5.5. Let $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ be a $\mathrm{N}_{eu}-$closed mapping and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping. Then their composition $gof:(X,\mathrm{T}_N)\to(Z,\eta_N)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

Proof. Let F be a $\mathrm{N}_{eu}-$closed set in X. Since f is $\mathrm{N}_{eu}-$closed, $f(F)$ is $\mathrm{N}_{eu}-$closed in Y. Since g is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed, $g[f(F)] = (gof)(F)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Z. Hence gof is a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping.

Theorem 5.6. Let $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be two mappings such that their composition $gof:(X,\mathrm{T}_N)\to(Z,\eta_N)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed. Then the following statements are true.

(i) If f is $\mathrm{N}_{eu}-$continuous and surjective, then g is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

(ii) If g is $\mathrm{N}_{eu}\alpha b^* g\alpha-$irresolute and injective, then f is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

Proof. (i) Let A be a $\mathrm{N}_{eu}-$closed set of Y. Since f is $\mathrm{N}_{eu}-$continuous, $f^{-1}(A)$ is $\mathrm{N}_{eu}-$closed in X. Since gof is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed, $(gof)(f^{-1}(A))$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Z. Since f is surjective, $g(A)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Z. Hence g is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

(ii) Let B be any $\mathrm{N}_{eu}-$closed set of X. Since gof is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed, $(gof)(B)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Z. Since g is $\mathrm{N}_{eu} gs\alpha^*-$irresolute, $g^{-1}(gof(B))$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Y. Since g is injective, $f(B)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Y. Hence f is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

Theorem 5.7. Let $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ be a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping.

(i) If A is $\mathrm{N}_{eu}-$closed set of X, then the restriction $f_A : A\to Y$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

(ii) If $A = f^{-1}(B)$ for some $\mathrm{N}_{eu}-$closed set B of Y, then the restriction $f_A : A\to Y$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed.

Proof. (i) Let B be any $\mathrm{N}_{eu}-$closed set of A. Then $B = A\cap F$ for some $\mathrm{N}_{eu}-$closed set F of X and so B is $\mathrm{N}_{eu}-$closed in X. By hypothesis, $f(B)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed in Y. But $f(B) = f_A(B)$, therefore f_A is a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping.

(ii) Let D be any $\mathrm{N}_{eu}-$closed set of A. Then $D = A\cap H$ for some $\mathrm{N}_{eu}-$closed set H in X. Now, $f_A(D) = f(D) = f(A\cap H) = f\left[f^{-1}(B)\cap H\right] = B\cap f(H)$. Since f is a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping, so $f(H)$ is a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed set in Y. Hence f_A is a $\mathrm{N}_{eu}\alpha b^* g\alpha-$closed mapping.

Theorem 5.8. A function $f:(X,\mathrm{T}_N)\to(Y,\sigma_N)$ is $\mathrm{N}_{eu}\alpha b^* g\alpha-$open if and only if $f\left[\mathrm{N}_{eu}Int(A)\right] \subseteq \mathrm{N}_{eu}\alpha b^* g\alpha-Int\left[f(A)\right]$, for every $\mathrm{N}_{eu}-$set A of X.

Proof. Suppose $f:(X,T_N)\to(Y,\sigma_N)$ is a $N_{eu}\alpha b*g\alpha-$open function and A is any $N_{eu}-$set in X. Then $N_{eu}Int(A)$ is a $N_{eu}-$open set in X. Since f $N_{eu}\alpha b*g\alpha-$open, $f[N_{eu}Int(A)]$ is a $N_{eu}\alpha b*g\alpha-$open set. Since $N_{eu}\alpha b*g\alpha-Int[f(N_{eu}IntA)]\subseteq N_{eu}\alpha b*g\alpha-Int[f(A)]$, $f[N_{eu}Int(A)]\subseteq N_{eu}\alpha b*g\alpha-Int[f(A)]$.

Conversely, $f[N_{eu}Int(A)]\subseteq N_{eu}\alpha b*g\alpha-Int[f(A)]$ for every $N_{eu}-$set A in X. Let U be a $N_{eu}-$open set in X. Then $N_{eu}Int(U)=U$ and by hypothesis, $f(U)\subseteq N_{eu}\alpha b*g\alpha-Int[f(U)]$. But $N_{eu}\alpha b*g\alpha-Int[f(U)]\subseteq f(U)$. Therefore, $f(U)=N_{eu}\alpha b*g\alpha-Int[f(U)]$. Then $f(U)$ is $N_{eu}\alpha b*g\alpha-$open. Hence f is a $N_{eu}\alpha b*g\alpha-$open mapping.

Theorem 5.9. A function $f:(X,T_N)\to(Y,\sigma_N)$ is $N_{eu}\alpha b*g\alpha-$open if and only if for each $x_{(r,s,t)}\in X$ and for each $N_{eu}-$neighbourhood U of $x_{(r,s,t)}$ in X, there exists a $N_{eu}\alpha b*g\alpha-$neighbourhood W of $f(x_{(r,s,t)})$ in Y such that $W\subseteq f(U)$.

Proof. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a $N_{eu}\alpha b*g\alpha-$open function. Let $x_{(r,s,t)}\in X$ and U be any arbitrary $N_{eu}-$neighbourhood of $x_{(r,s,t)}$ in X. Then there exists a $N_{eu}-$open set G such that $x_{(r,s,t)}\in G\subseteq U$. By Theorem 5.8, $f(G)=f[N_{eu}Int(G)]\subseteq N_{eu}\alpha b*g\alpha-Int[f(G)]$. But, $N_{eu}\alpha b*g\alpha-Int[f(G)]\subseteq f(G)$. Therefore, $N_{eu}\alpha b*g\alpha-Int[f(G)]=f(G)$ and hence $f(G)$ is $N_{eu}\alpha b*g\alpha-$open in Y. Since $x_{(r,s,t)}\in G\subseteq U$, $f(x_{(r,s,t)})\in f(G)\subseteq f(U)$ and so the result follows by taking $W=f(G)$.

Conversely, Let U be any $N_{eu}-$open set in X. Let $x_{(r,s,t)}\in U$ and $f(x_{(r,s,t)})=y_{(k,l,m)}$. Then by assumption there exists a $N_{eu}\alpha b*g\alpha-$neighbourhood $W_{(y_{(k,l,m)})}$ of $y_{(k,l,m)}$ in Y such that $W_{(y_{(k,l,m)})}\subseteq f(U)$. Since $W_{(y_{(k,l,m)})}$ is a $N_{eu}\alpha b*g\alpha-$neighbourhood of $y_{(k,l,m)}$, there exists a $N_{eu}\alpha b*g\alpha-$open set $V_{(y_{(k,l,m)})}$ in Y such that $y_{(k,l,m)}\in V_{(y_{(k,l,m)})}\subseteq W_{(y_{(k,l,m)})}$. Therefore, $f(U)=\bigcup\left\{V_{(y_{(k,l,m)})}:y_{(k,l,m)}\in f(U)\right\}$. Since the union of $N_{eu}\alpha b*g\alpha-$open sets is $N_{eu}\alpha b*g\alpha-$open, $f(U)$ is a $N_{eu}\alpha b*g\alpha-$open set in Y. Thus, f is a $N_{eu}\alpha b*g\alpha-$open mapping.

Theorem 5.10. For any bijective mapping $f:(X,T_N)\to(Y,\sigma_N)$ the following statements are equivalent:

(i) $f^{-1}:Y\to X$ is $N_{eu}\alpha b*g\alpha-$continuous.

(ii) f is $N_{eu}\alpha b*g\alpha-$open.

(iii) f is $N_{eu}\alpha b*g\alpha-$closed.

Proof. $(\mathrm{i})\Rightarrow(\mathrm{ii})$: Let U be a N_{eu}-open set in X. By assumption, $\left(f^{-1}\right)^{-1}(U)=f(U)$ is $N_{eu}\alpha b^{*}g\alpha$-open in Y and so f is $N_{eu}\alpha b^{*}g\alpha$-open.

$(\mathrm{ii})\Rightarrow(\mathrm{iii})$: Let F be a N_{eu}-closed set of X. Then F^{c} is a N_{eu}-open set in X. By assumption $f(F^{c})$ is $N_{eu}\alpha b^{*}g\alpha$-open in Y. But $f(F^{c})=\left[f(F)\right]^{c}$. Therefore $f(F)$ is $N_{eu}\alpha b^{*}g\alpha$-closed set in Y. Hence, f is $N_{eu}\alpha b^{*}g\alpha$-closed.

$(\mathrm{iii})\Rightarrow(\mathrm{i})$: Let F be a N_{eu}-closed set of X. By assumption, $f(F)$ is $N_{eu}\alpha b^{*}g\alpha$-closed set in Y. But $f(F)=\left(f^{-1}\right)^{-1}(F)$ and therefore by Theorem 3.4, $f^{-1}:Y\rightarrow X$ is $N_{eu}\alpha b^{*}g\alpha$-continuous.

6 Strongly Neutrosophic $\alpha b^{*}g\alpha$-Continuous Mappings and Perfectly Neutrosophic $\alpha b^{*}g\alpha$-Continuous Mappings

In this section, we introduce and study the concepts of strongly $N_{eu}\alpha b^{*}g\alpha$-continuous and perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous mappings in N_{eu}-*Top*-*Spaces*.

Definition 6.1. A mapping $f:(X,T_{N})\rightarrow(Y,\sigma_{N})$ is called strongly $N_{eu}\alpha b^{*}g\alpha$-continuous if the inverse image of every $N_{eu}\alpha b^{*}g\alpha$-open set in Y is N_{eu}-open in X.

Definition 6.2. A mapping $f:(X,T_{N})\rightarrow(Y,\sigma_{N})$ is called perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous if the inverse image of every $N_{eu}\alpha b^{*}g\alpha$-open set in Y is N_{eu}-clopen in X.

Theorem 6.3. Let $f:(X,T_{N})\rightarrow(Y,\sigma_{N})$ be a mapping. Then the following statements are true:

(i) If f is perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous, then f is perfectly N_{eu}-continuous.

(ii) If f is strongly $N_{eu}\alpha b^{*}g\alpha$-continuous, then f is N_{eu}-continuous.

Proof. (i) Let $f:X\rightarrow Y$ be perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous. Let V be a N_{eu}-open set in Y. Then V is $N_{eu}\alpha b^{*}g\alpha$-open set in Y. Since f is perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous, $f^{-1}(V)$ is N_{eu}-clopen in X. Hence f is perfectly N_{eu}-continuous.

(ii) Let $f:X\rightarrow Y$ be strongly $N_{eu}\alpha b^{*}g\alpha$-continuous. Let G be a N_{eu}-open set in Y. Then G is $N_{eu}\alpha b^{*}g\alpha$-open set in Y. Since f is strongly $N_{eu}gs\alpha^{*}$-continuous, $f^{-1}(G)$ is N_{eu}-open in X. Therefore f is N_{eu}-continuous.

Theorem 6.4. Let $f:X\rightarrow Y$ be strongly $N_{eu}\alpha b^{*}g\alpha$-continuous and A be N_{eu}-open set in X. Then the restriction map, $f_{A}:A\rightarrow Y$ is strongly $N_{eu}\alpha b^{*}g\alpha$-continuous.

Proof. Let V be any $N_{eu}\alpha b^{*}g\alpha$-open set in Y. Since f is strongly $N_{eu}gs\alpha^{*}$-continuous, $f^{-1}(V)$ is N_{eu}-open in X. But $f_{A}^{-1}(V)=A\cap f^{-1}(V)$. Since A and $f^{-1}(V)$ are N_{eu}-open, $f_{A}^{-1}(V)$ is N_{eu}-open in A. Hence f_{A} is strongly $N_{eu}gs\alpha^{*}$-continuous.

Theorem 6.5. Every perfectly $N_{eu}\alpha b^*g\alpha$-continuous mapping $f:(X,T_N)\to(Y,\sigma_N)$ is strongly $N_{eu}\alpha b^*g\alpha$-continuous.

Proof. Let $f:X\to Y$ be perfectly $N_{eu}\alpha b^*g\alpha$-continuous and V be $N_{eu}\alpha b^*g\alpha$-open set in Y. Since f is perfectly $N_{eu}\alpha b^*g\alpha$-continuous, $f^{-1}(V)$ is N_{eu}-clopen in X. That is both N_{eu}-open and N_{eu}-closed in X. Hence f is strongly $N_{eu}\alpha b^*g\alpha$-continuous.

Theorem 6.6. If $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ are strongly $N_{eu}\alpha b^*g\alpha$-continuous, then $gof:(X,T_N)\to(Z,\eta_N)$ is also strongly $N_{eu}\alpha b^*g\alpha$-continuous.

Proof. Let V be a N_{eu}-open set in Z. Since g is a strongly $N_{eu}\alpha b^*g\alpha$-continuous mapping, $g^{-1}(V)$ is N_{eu}-open in Y. Then $g^{-1}(V)$ is N_{eu}-open in Y. Since f is a strongly $N_{eu}\alpha b^*g\alpha$-continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is N_{eu}-open in X. Therefore, gof is strongly $N_{eu}\alpha b^*g\alpha$-continuous.

Theorem 6.7. If $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ are perfectly $N_{eu}\alpha b^*g\alpha$-continuous mappings, then their composition $gof:(X,T_N)\to(Z,\eta_N)$ is also perfectly $N_{eu}\alpha b^*g\alpha$-continuous mapping.

Proof. Let V be a $N_{eu}\alpha b^*g\alpha$-open set in Z. Since g is a perfectly $N_{eu}gs\alpha^*$-continuous mapping, $g^{-1}(V)$ is N_{eu}-clopen in Y. That is $g^{-1}(V)$ both N_{eu}-open and N_{eu}-closed in X. Then $g^{-1}(V)$ is $N_{eu}\alpha b^*g\alpha$-open set in X. Since f is a perfectly $N_{eu}gs\alpha^*$-continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is N_{eu}-clopen in X. Therefore gof is perfectly $N_{eu}\alpha b^*g\alpha$-continuous.

Theorem 6.8. Let $f:(X,T_N)\to(Y,\sigma_N)$ and $g:(Y,\sigma_N)\to(Z,\eta_N)$ be mappings. Then the following statements are true.

(i) If g is strongly $N_{eu}\alpha b^*g\alpha$-continuous and f is $N_{eu}\alpha b^*g\alpha$-continuous, then gof is $N_{eu}\alpha b^*g\alpha$-irresolute.

(ii) If g is perfectly $N_{eu}\alpha b^*g\alpha$-continuous and f is N_{eu}-continuous, then gof is strongly $N_{eu}\alpha b^*g\alpha$-continuous.

(iii) If g is strongly $N_{eu}\alpha b^*g\alpha$-continuous and f is perfectly $N_{eu}\alpha b^*g\alpha$-continuous, then gof is perfectly $N_{eu}\alpha b^*g\alpha$-continuous.

(iv) If g is $N_{eu}\alpha b^*g\alpha$-continuous and f is strongly $N_{eu}\alpha b^*g\alpha$-continuous, then gof is N_{eu}-continuous.

Proof. (i) Let V be a $N_{eu}\alpha b^*g\alpha$-open set in Z. Since g is a strongly $N_{eu}\alpha b^*g\alpha$-continuous mapping, $g^{-1}(V)$ is N_{eu}-open set in Y. Since f is a $N_{eu}\alpha b^*g\alpha$-continuous mapping,

$f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is $N_{eu}\alpha b^{*}g\alpha$-open set in X. Hence gof is $N_{eu}\alpha b^{*}g\alpha$-irresolute.

(ii) Let V be a $N_{eu}\alpha b^{*}g\alpha$-open set in Z. Since g is a perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous mapping, $g^{-1}(V)$ is N_{eu}-clopen set in Y. That is, $g^{-1}(V)$ is both N_{eu}-open and N_{eu}-closed. Since f is a $N_{eu}\alpha b^{*}g\alpha$-continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is N_{eu}-open in X. Therefore gof is strongly $N_{eu}\alpha b^{*}g\alpha$-continuous.

(iii) Let V be a $N_{eu}\alpha b^{*}g\alpha$-open set in Z. Since g is a strongly $N_{eu}\alpha b^{*}g\alpha$-continuous mapping, $g^{-1}(V)$ is N_{eu}-open set in Y. Since every N_{eu}-open set is $N_{eu}\alpha b^{*}g\alpha$-open set. So $g^{-1}(V)$ is $N_{eu}\alpha b^{*}g\alpha$-open set in X. Since f is a perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous mapping, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is N_{eu}-clopen in X. Hence gof is perfectly $N_{eu}\alpha b^{*}g\alpha$-continuous.

(iv) Let V be a N_{eu}-open set in Z. Since g is a $N_{eu}\alpha b^{*}g\alpha$-continuous mapping, $g^{-1}(V)$ is $N_{eu}\alpha b^{*}g\alpha$-open set in Y. Since f is a strongly $N_{eu}\alpha b^{*}g\alpha$-continuous map, $f^{-1}\left[g^{-1}(V)\right]=(gof)^{-1}(V)$ is N_{eu}-open in X. So gof is N_{eu}-continuous.

7 Contra Neutrosophic $\alpha b^{*}g\alpha$-Continuous Mappings and Contra Neutrosophic $\alpha b^{*}g\alpha$-Irresolute Mappings

In this section, we introduce the concepts of contra $N_{eu}\alpha b^{*}g\alpha$-continuous mappings and contra $N_{eu}\alpha b^{*}g\alpha$-irresolute mappings and investigate their fundamental properties and characterizations.

Definition 7.1. A mapping $f:(X,T_N)\to(Y,\sigma_N)$ is said to be contra N_{eu}-continuous if the inverse image of every N_{eu}-open set in Y is N_{eu}-closed set in X.

Definition 7.2. A mapping $f:(X,T_N)\to(Y,\sigma_N)$ is called contra $N_{eu}\alpha b^{*}g\alpha$-continuous if the inverse image of every N_{eu}-open set in Y is $N_{eu}\alpha b^{*}g\alpha$-closed in X.

Theorem 7.3. Let $f:(X,T_N)\to(Y,\sigma_N)$ be a contra N_{eu}-continuous mapping. Then f is contra $N_{eu}\alpha b^{*}g\alpha$-continuous.

Proof. Let V be any N_{eu}-open set in Y. Since f is contra N_{eu}-continuous, $f^{-1}(V)$ is N_{eu}-closed set in X. As every N_{eu}-closed set is $N_{eu}\alpha b^{*}g\alpha$-closed, we have $f^{-1}(V)$ is $N_{eu}\alpha b^{*}g\alpha$-closed set in X. Therefore f is contra $N_{eu}\alpha b^{*}g\alpha$-continuous.

Theorem 7.4. A mapping $f:(X,T_N)\to(Y,\sigma_N)$ is contra $N_{eu}\alpha b^{*}g\alpha$-continuous if and only if the inverse image of every N_{eu}-closed set in Y is $N_{eu}\alpha b^{*}g\alpha$-open set in X.

Proof. Let V be a N_{eu} –closed set in Y. Then V^C is N_{eu} –open set in Y. Since f is contra $N_{eu}\alpha b^* g\alpha$ –continuous, $f^{-1}(V^C)$ is $N_{eu}\alpha b^* g\alpha$ –closed set in X. But $f^{-1}(V^C) = 1 - f^{-1}(V)$ and so $f^{-1}(V)$ is $N_{eu}\alpha b^* g\alpha$ –open set in X. Conversely, assume that the inverse image of every N_{eu} –closed set in Y is $N_{eu}\alpha b^* g\alpha$ –open in X. Let W be a N_{eu} –open set in Y. Then W^C is N_{eu} –closed in Y. By hypothesis $f^{-1}(W^C) = 1 - f^{-1}(W)$ is $N_{eu}\alpha b^* g\alpha$ –open in X, and so $f^{-1}(W)$ is $N_{eu}\alpha b^* g\alpha$ –closed set in X. Thus f is contra $N_{eu} gs\alpha^*$ –continuous.

Theorem 7.5. If a mapping $f:(X,T_N) \to (Y,\sigma_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –continuous and $g:(Y,\sigma_N) \to (Z,\eta_N)$ is N_{eu} –continuous, then their composition $gof:(X,T_N) \to (Z,\eta_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –continuous.

Proof. Let W ba a N_{eu} –open set in Z. Since g is N_{eu} –continuous, $g^{-1}(W)$ is N_{eu} –open set in Y. Since f is contra $N_{eu}\alpha b^* g\alpha$ –continuous, $f^{-1}\left[g^{-1}(W)\right]$ is $N_{eu}\alpha b^* g\alpha$ –closed set in X. But $(gof)^{-1}(W) = f^{-1}\left[g^{-1}(W)\right]$. Thus gof is contra $N_{eu}\alpha b^* g\alpha$ –continuous.

Definition 7.6. A mapping $f:(X,T_N) \to (Y,\sigma_N)$ is called contra $N_{eu}\alpha b^* g\alpha$ –irresolute if the inverse image of every $N_{eu}\alpha b^* g\alpha$ –open set in Y is $N_{eu}\alpha b^* g\alpha$ –closed in X.

Theorem 7.7. If a mapping $f:(X,T_N) \to (Y,\sigma_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –irresolute, then it is contra $N_{eu}\alpha b^* g\alpha$ –continuous.

Proof. Let V be a N_{eu} –open set in Y. Since every N_{eu} –open set is $N_{eu}\alpha b^* g\alpha$ –open, V is N_{eu} –open set in Y. Since f is contra $N_{eu}\alpha b^* g\alpha$ –irresolute, $f^{-1}(V)$ is $N_{eu}\alpha b^* g\alpha$ –closed set in X. Thus f is contra $N_{eu}\alpha b^* g\alpha$ –continuous.

Theorem 7.8. Let (X,T_N), (Y,σ_N) and (Z,η_N) be N_{eu} *–Top –Spaces*. If $f:(X,T_N) \to (Y,\sigma_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –irresolute and $g:(Y,\sigma_N) \to (Z,\eta_N)$ is $N_{eu}\alpha b^* g\alpha$ –continuous, then $gof:(X,T_N) \to (Z,\eta_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –continuous.

Proof. Let W be any N_{eu} –open set in Z. Since g is $N_{eu}\alpha b^* g\alpha$ –continuous, $g^{-1}(W)$ is $N_{eu}\alpha b^* g\alpha$ –open set in Y. Since f is contra $N_{eu}\alpha b^* g\alpha$ –irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is $N_{eu}\alpha b^* g\alpha$ –closed set in X. But $(gof)^{-1}(W) = f^{-1}\left[g^{-1}(W)\right]$. Thus gof is contra $N_{eu}\alpha b^* g\alpha$ ––continuous.

Theorem 7.9. If $f:(X,T_N) \to (Y,\sigma_N)$ is $N_{eu}\alpha b^* g\alpha$ –irresolute and $g:(Y,\sigma_N) \to (Z,\eta_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –irresolute, then their composition $gof:(X,T_N) \to (Z,\eta_N)$ is contra $N_{eu}\alpha b^* g\alpha$ –irresolute mapping.

Proof. Let W be any $N_{eu}\alpha b^*g\alpha$-open set in Z. Since g is contra $N_{eu}\alpha b^*g\alpha$-irresolute, $g^{-1}(W)$ is $N_{eu}\alpha b^*g\alpha$-closed set in Y. Since f is $N_{eu}\alpha b^*g\alpha$-irresolute, $f^{-1}\left[g^{-1}(W)\right]$ is $N_{eu}\alpha b^*g\alpha$-closed set in X. But $(gof)^{-1}(W) = f^{-1}\left[g^{-1}(W)\right]$. Thus gof is contra $N_{eu}\alpha b^*g\alpha$-irresolute.

Conclusion

In this research article, we have introduced and studied the properties of $N_{eu}\alpha b^*g\alpha$-continuous functions, $N_{eu}\alpha b^*g\alpha$-irresolute functions, $N_{eu}gs\alpha^*$-closed functions, $N_{eu}\alpha b^*g\alpha$-open functions, strongly $N_{eu}\alpha b^*g\alpha$-continuous functions, perfectly $N_{eu}\alpha b^*g\alpha$-continuous functions, contra $N_{eu}\alpha b^*g\alpha$-continuous functions, and contra $N_{eu}\alpha b^*g\alpha$-irresolute functions in N_{eu}-*Top*-*Spaces* and established the relations between them. We have obtained fundamental characterizations of theses mappings and investigated preservation properties. We expect the results in this article will be basis for further applications of mappings in N_{eu}-*Top*-*Spaces*.

Recommendations

It is recommended to introduce $N_{eu}\alpha b^*g\alpha$-compactness, $N_{eu}\alpha b^*g\alpha$-connectedness, $N_{eu}\alpha b^*g\alpha$-regular spaces, and $N_{eu}\alpha b^*g\alpha$-normal spaces in N_{eu}-*Top*-*Spaces* and investigate their fundmental properties and characterizations.

Scientific Ethics Declaration

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Conflict of Interest

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