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# Studying the Temperature Effect on Lengthwise Fracture of Inhomogeneous Beam Subjected to Impact Loading

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**Abstract**: The wide use of continuously inhomogeneous engineering materials in various load-bearing structures puts high requirements with respect to their fracture behavior. In many applications fracture represents the most often mode of structural failure. In view of this, exploring fracture behavior under various loading conditions is highly needed for guaranteeing the load-bearing capacity, safety and proper functioning of engineering structures. Developing of analyses which encompass more factors influencing fracture behavior of continuously inhomogeneous structures is very important direction in fracture mechanics. One of the extreme loadings acting on engineering structures is the impact loading which may be generated by objects falling on the structure. It happens frequently that impact loading acts in combination with other influences on the structure like, for instance, temperature field. Therefore, this paper deals with analysis of the effect of temperature filed on the lengthwise fracture in a continuously inhomogeneous beam structure subjected to impact loading by an object falling on the structure. The beam has non-linear elastic behavior. The fracture under impact loading and temperature field is analyzed by considering the strain energy release rate. The analysis is verified by applying the integral *J*. The change of the fracture behavior due to temperature field and the parameters of the impact loading is studied.

**Keywords:** Continuously inhomogeneous material, Impact, Fracture, Temperature, Beam structure

## Introduction

The rapid advance in the field of technologies for manufacturing of modern continuously inhomogeneous structural materials that has been made in the recent decades across the globe facilitates intensification of the application of these materials in various segments of current engineering (Gandra et al., 2011, El-Galy et al., 2019). Continuously inhomogeneous composites like functionally graded materials are attracting the attention of academicians and engineers in many countries due to their advantages in comparison to the homogeneous structural materials (Mahamood & Akinlabi, 2017). One of the main advantages is the smooth change of material properties along certain directions in the solid (Miyamoto et al., 1999, Radhika et al., 2020). Besides, this change can be controlled in the process of manufacturing in order to satisfy particular requirements with respect to strength, load-carrying capacity, stiffness, aging, fatigue, stability, etc. (Mahamood & Akinlabi, 2017, Radhika et al., 2020).

The engineering practice shows that increasing the use of continuous inhomogeneous materials and structures in a variety of applications leads to a stronger interest towards their fracture behavior in different conditions of work and loadings. This is due to the fact that namely the fracture behavior has a decisive meaning for reliability and safety of inhomogeneous engineering structures. Besides, in many situations fracture is one of the main causes for failure of structural members (Dowling, 2007, Rizov, 2018, Rizov & Altenbach, 2019). Also, fracture may result in sudden collapse of entire engineering facilities with heavy consequences in terms of financial losses and even human lives.

In this paper, we study lengthwise fracture in a continuously inhomogeneous beam structure that is subjected to impact loading. The latter is generated by an object that falls on the structure. Besides the impact loading, the beam is under temperature field. In fact, the main goal of this paper is to analyze the effect of temperature field on the lengthwise fracture in the beam subjected to impact loading. The interest towards fracture behavior under impact loading and temperature field is provoked by the fact that continuously inhomogeneous structural members frequently are used in applications where they are under combined influence of loading and temperature. The beam considered in this paper has non-linear elastic behavior. Besides, the beam has continuous material inhomogeneity in both transversal and longitudinal directions. When deriving the strain energy release rate in the beam, both impact loading and the effect of temperature field are taken into account. The integral *J* is used for verification of the strain energy release rate. The change of the strain energy release rate is induced by the temperature field and is evaluated. The effects of the parameters of the impact loading and the material inhomogeneity on the strain energy release rate are also assessed.

## Analysis

This paper deals with lengthwise fracture in the continuously inhomogeneous beam displayed in Figure 1.

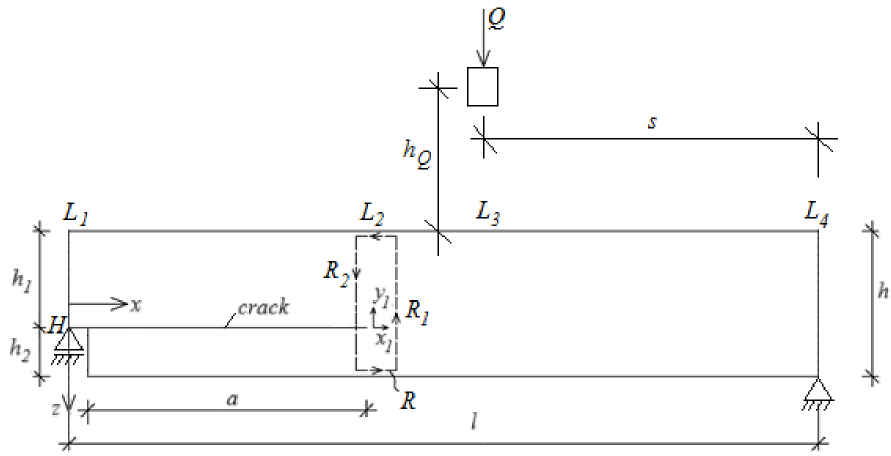


Figure 1. Beam structure under impact loading

The beam is simply supported in its end. A lengthwise crack of length, $a$, is located in portion, $L_1L_2$, of the beam. The thickness of the upper arm of the crack is $h_1$. The lower arm of the crack has thickness, $h_2$, and is free of stresses since the support is located in point, $H$, of the upper arm of the crack (Figure 1). The beam is under impact by an object of weight, $Q$. This object is dropped from height, $h_Q$, and hits the beam in point, $L_3$, as displayed in Figure 1. Besides the impact loading, the beam is also under temperature field (the temperature change is $T$ ).

The beam has non-linear elastic behavior is treated by the constitutive law in Eq. (1) (Lukash, 1997).

$$\sigma = (E\varepsilon - B\varepsilon^m)\left(1 + D\ln\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right), \tag{1}$$

where $E$, $B$, $m$, $D$ and $\dot{\varepsilon}_0$ are parameters, $\sigma$ is the stress, $\varepsilon$ is the strain, $\dot{\varepsilon}$ is the speed.

The beam is continuously inhomogeneous along the thickness and length. Because of this, the parameters, $E$, $B$, $m$ and $D$, change smoothly both transversely and longitudinally of the beam structure. The change of these parameters transversely of the beam is described by the laws in Eqs. (2), (3), (4) and (5).

$$E = E_1 e^{\beta_1 \frac{\frac{h}{2}+z}{h}}, \tag{2}$$

$$B = B_1 e^{\beta_2 \frac{\frac{h}{2}+z}{h}}, \tag{3}$$

$$m = m_1 e^{\beta_3 \frac{\frac{h}{2}+z}{h}}, \tag{4}$$

$$D = D_1 e^{\beta_4 \frac{\frac{h}{2}+z}{h}}, \tag{5}$$

where

$$-\frac{h}{2} \leq z \leq \frac{h}{2}. \tag{6}$$

Here, $E_1$, $B_1$, $m_1$ and $D_1$ are the values of $E$, $B$, $m$ and $D$ on the upper surface of the beam, $\beta_1$, $\beta_2$, $\beta_3$ and $\beta_4$ are parameters, $h$, is the beam thickness, $z$ is the vertical axis of the beam cross-section. The quantities, $E_1$, $B_1$, $m_1$ and $D_1$, change along the beam length according to the laws in Eqs. (7), (8), (9) and (10).

$$E_1 = E_2 e^{\beta_5 \frac{x}{l}}, \tag{7}$$

$$B_1 = B_2 e^{\beta_6 \frac{x}{l}}, \tag{8}$$

$$m_1 = m_2 e^{\beta_7 \frac{x}{l}}, \tag{9}$$

$$D_1 = D_2 e^{\beta_8 \frac{x}{l}}, \tag{10}$$

where

$$0 \leq x \leq l. \tag{11}$$

In the above formulas, $E_2$, $B_2$, $m_2$ and $D_2$ are the values of $E_1$, $B_1$, $m_1$ and $D_1$ in the left-hand end of the beam structure, $\beta_5$, $\beta_6$, $\beta_7$ and $\beta_8$ are parameters, $l$ is the beam length, $x$ is the longitudinal axis of the beam. In order to analyze the lengthwise fracture under impact loading, first we have to determine the static vertical displacement, $\delta_{L3st}$, of point $L_3$ (this is the displacement induced by a vertical force of magnitude, $Q$, applied statically in $L_3$). Equation (12) is applied for determining of $\delta_{L3st}$.

$$\delta_{L3st} = \sum \int \kappa_i M_i dx, \tag{12}$$

where $\kappa_i$ and $M_i$ are the curvature and the moment.

The curvature is related to the change of the strain along the thickness via Eq. (13).

$$\varepsilon = \kappa_i (z - z_n), \tag{13}$$

where

$$-\frac{h}{2} \leq z \leq \frac{h}{2}. \tag{14}$$

In Eq. (13), $z_n$ is the neutral axis coordinate.

The conditions of equilibrium written in Eqs. (15) and (16) are applied for determining of the curvature and the neutral axis coordinate.

$$N = \iint_{(A)} \sigma dA, \tag{15}$$

$$M = \iint_{(A)} \sigma z dA, \tag{16}$$

where $N$ and $M$ are the axial force and the bending moment, $A$ is the beam cross-section area, $\sigma$ is the stress obtained by Eq. (1).

When determining the curvature and the neutral axis coordinate, the effect of temperature field in the beam structure is treated by changing of $E$ through Eqs. (17) and (18) (Narisawa, 1987).

$$E_{T_0}\left(\frac{t}{\alpha_T}\right) = \frac{T_0 r_0}{T r_1} E, \tag{17}$$

$$\lg \alpha_T = -\frac{C_P\left(T - T_p\right)}{C_{RF} + \left(T - T_p\right)}, \tag{18}$$

where $t$ is the time, $E_{T_0}$ is the value of $E$ at room temperature, the ratio, $T_0 r_0 /(T r_1)$, is unit, $T_p$ is a material parameter, $C_P = 8.86$ and $C_{RF} = 101.6$ (Narisawa, 1987).

It is clear that the axial force in each portion of the beam is zero (Figure 1). The bending moment is found by using Eq. (19).

$$M = Q\frac{s}{l}x, \tag{19}$$

where $s$ is the distance between points, $L_3$ and $L_4$, as displayed in Figure 1. Equation (19) holds in portion, $L_1 L_3$, of the beam, i.e.

$$0 \le x \le l - s. \tag{20}$$

The bending moment in portion, $L_3 L_4$, is determined by Eq. (21).

$$M = Q\frac{s}{l}x - Q\left(x - l + s\right), \tag{21}$$

where

$$l - s \le x \le l. \tag{22}$$

Equations (15) and (16) are solved by the MatLab to determine the curvatures and the coordinates of the neutral axis in portions, $L_1 L_2$, $L_2 L_3$ and $L_3 L_4$, of the beam structure.

The displacement, $\delta_{L3st}$, is applied to determine the dynamic coefficient, $k_{dyn}$. By using the methodology taken from (Ickovitch et al., 1997), we obtain

$$k_{dyn} = 1 + \sqrt{1 + 2\frac{h_Q}{\delta_{L3st}}\frac{Q}{Q + 0.487 G_{str}}}, \tag{23}$$

where $G_{str}$ is the structure deadweight.

The dynamic stress, $\sigma_{dyni}$, in an arbitrary portion of the beam is found by Eq. (24).

$$\sigma_{dyni} = k_{dyn}\sigma_{sti}, \tag{24}$$

where $\sigma_{sti}$ is the stress induced by the static loading of the beam structure by force, $Q$.

The complementary strain energy density, $u_{0i}^{*}$, is derived by using Eq. (25).

$$u_{0i}^{*} = \sigma_{dyni}\,\varepsilon_i - \int \sigma_{dyni} d\varepsilon \,. \tag{25}$$

Equation (26) is used to calculate the strain energy release rate, $G$.

$$G = \frac{dU^{*}}{bda}, \tag{26}$$

where $U^{*}$ is the complementary strain energy in the beam, $b$ is the cross-section width. This energy is found by Eq. (27).

$$U^{*} = U_{L1L2}^{*} + U_{L2L3}^{*} + U_{L3L4}^{*}, \tag{27}$$

where

$$U_{L1L2}^{*} = \iiint\limits_{(V_{L1L2})} u_{01}^{*} dV \,, \tag{28}$$

$$U_{L2L3}^{*} = \iiint\limits_{(V_{L2L3})} u_{02}^{*} dV \,, \tag{29}$$

$$U_{L3L4}^{*} = \iiint\limits_{(V_{L3L4})} u_{03}^{*} dV \,. \tag{30}$$

Here, $V_{L1L2}$, $V_{L2L3}$ and $V_{L3L4}$ are the volumes of the upper arm of the crack, and portions, $L_2L_3$ and $L_3L_4$, of the beam, respectively.

The analysis of the strain energy release rate is checked-up by applying the integral *J* (Broek, 1986). The contour, $R$, is used for carrying-out the integration (Figure 1). In this case, the expression for the integral *J* has two members, $J_{R1}$ and $J_{R2}$, as written in Eq. (31).

$$J = J_{R1} + J_{R2}, \tag{31}$$

where $J_{R1}$ and $J_{R2}$ are related to portions, $R_1$ and $R_2$, of $R$.

The two members are given in Eqs. (32) and (33), respectively.

$$J_{R1} = \int \left[ u_{0R1} \cos\alpha_{R1} - \left( p_{x1} \frac{\partial u_{R1}}{\partial x_1} + p_{y1} \frac{\partial v_{R1}}{\partial x_1} \right) \right] ds \,, \tag{32}$$

$$J_{R2} = \int \left[ u_{0R2} \cos\alpha_{R2} - \left( p_{x2} \frac{\partial u_{R2}}{\partial x_1} + p_{y2} \frac{\partial v_{R2}}{\partial x_1} \right) \right] ds \,. \tag{33}$$

The quantities involved in Eqs. (32) and (33) are determined by using the dynamic stresses, strains and displacements in the beam structure induced by the impact loading with taking into account the effect of temperature field. The MatLab is applied for integration in Eqs. (32) and (33). Since the integral *J* agrees with the strain energy release rate, we can make conclusion that the analysis is correct.

## Results of Parametric Investigation

Curves revealing how the lengthwise fracture in the non-linear elastic beam under impact loading is affected by the temperature field are reported in this section of the paper. The effects of the parameters of impact loading, the inhomogeneity and the beam structure geometry are also studied. The curves reported here are for a beam structure with $b = 0.015$ m, $h = 0.028$ m, $l = 0.900$ m, $a = 0.400$ m, $s = 0.400$ m and $h_1 = 0.010$ m.

One can observe the influence of the temperature field on the lengthwise fracture in Figure 2 where the strain energy release rate in non-dimensional form is presented as a function of parameter, $\beta_1$, at $T/T_P = 1.15$ (curve 1), $T/T_P = 1.30$ (curve 2) and $T/T_P = 1.45$ (curve 3). The curves in Figure 2 reveal that the strain energy release rate under impact loading reduces when $T/T_p$ ratio increases. The rise of the value of the parameter, $\beta_1$, also induces reduction of the strain energy release rate (Figure 2).

Figure 3 gives one idea about the influence of the value of the parameter, $\beta_5$, on the lengthwise fracture at $l/b = 20$ (curve 1), $l/b = 40$ (curve 2) and $l/b = 60$ (curve 3). The ratio, $l/b$, characterizes the beam span. $\beta_5$ is the parameter of the change of $E$ along the beam length. By inspecting of curves reported in Figure 3 one can notice that the strain energy release rate reduces when the parameter, $\beta_5$, increases its value. The growth of $l/b$ ratio results in increase of the strain energy release rate (Figure 3).

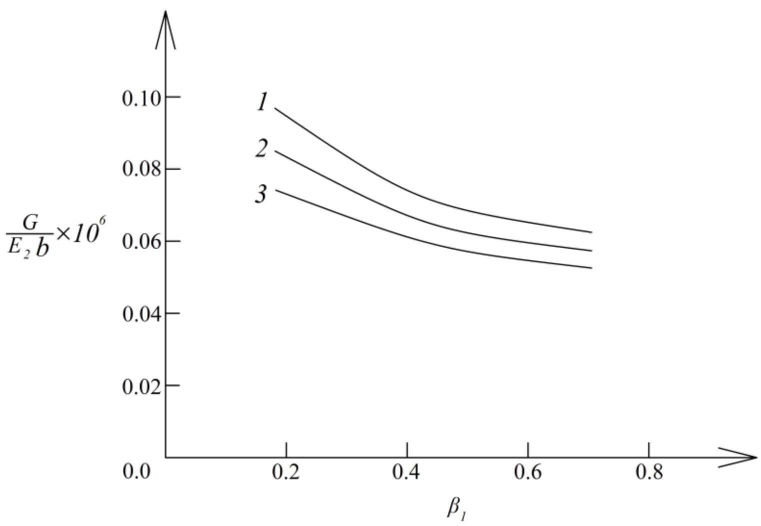


Figure 2. Strain energy release rate versus $\beta_1$

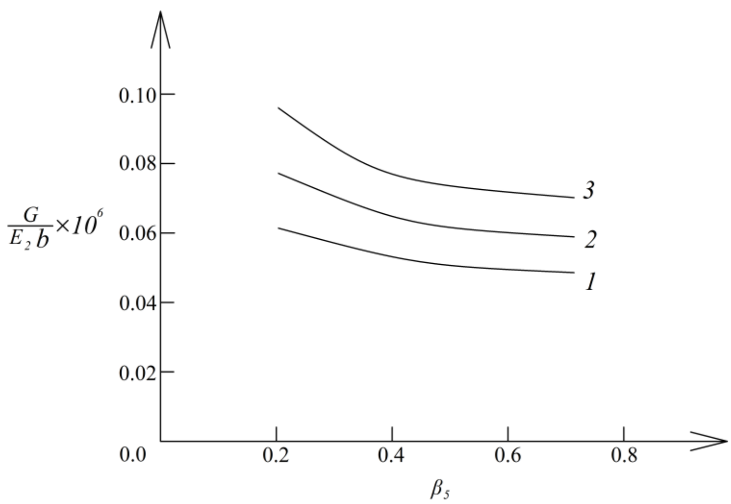


Figure 3. Strain energy release rate versus $\beta_5$

The impact loading is characterized by $h_Q / l$ ratio. In order to get an idea about the effect of the impact loading on the lengthwise fracture, we present the strain energy release rate as a function of the parameter, $\beta_2$, at $h_Q / l = 0.2$ (curve 1), $h_Q / l = 0.4$ (curve 2) and $h_Q / l = 0.6$ (curve 3) in Figure 4. $\beta_2$ is the parameter of the change of $B$ across the beam thickness. The conclusion that can be made by observing the curves reported in Figure 4 is that the strain energy release rate quickly rises with growth of $h_Q / l$ ratio (this is due to rise of the impact energy). The growth of the parameter, $\beta_2$, generates also rise of the strain energy release rate as can be observed in Figure 4.

Finally, we explore how the lengthwise fracture under impact loading is influenced by parameters, $\beta_4$ and $\beta_8$. As already mentioned, these parameters are related to the change of $D$ along the thickness and length of the beam structure. The exploration leads to curves reported in Figure 5 where the strain energy release rate is presented as a function of $\beta_4$ at $\beta_8 = 0.3$ (curve 1), $\beta_8 = 0.5$ (curve 2) and $\beta_8 = 0.7$ (curve 3).

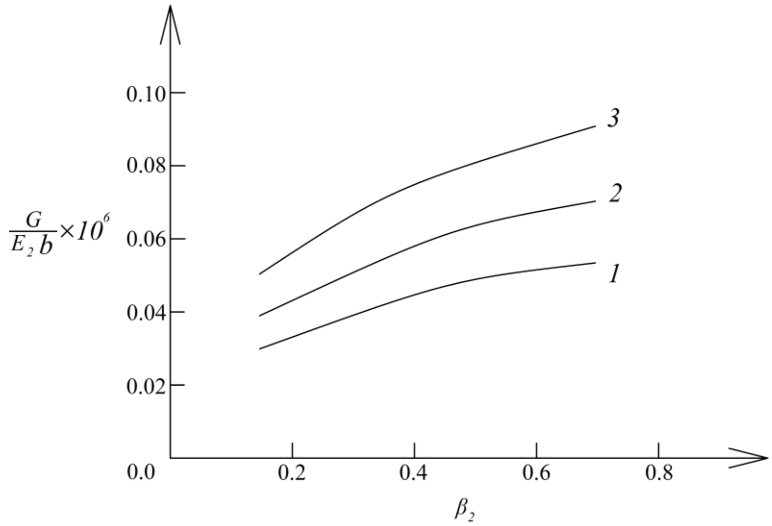


Figure 4. Strain energy release rate versus $\beta_2$

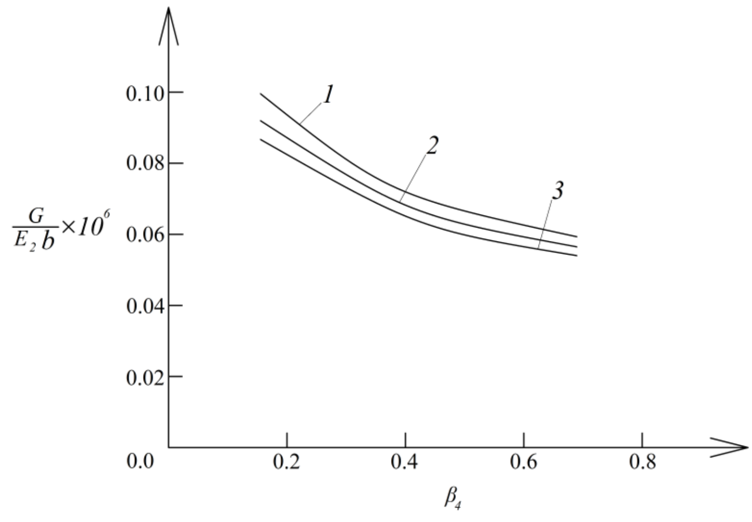


Figure 5. Strain energy release rate versus $\beta_4$

Inspection of curves in Figure 5 shows that the strain energy release rate reduces when the parameters, $\beta_4$ and $\beta_8$, increase.

## Conclusion

It is analyzed how the lengthwise fracture in a non-linear elastic beam structure subjected to impact loading is affected by a temperature field. The beam under consideration is continuously inhomogeneous along its thickness and length. The strain energy release rate in the impacted beam is solved. The solution derived accounts for the effect of the temperature field. A check-up is performed by using the integral *J*. The study indicates that the strain energy release rate reduces with increase of the $T/T_p$ ratio (this ratio characterizes the temperature field in the beam structure). It is found also that the strain energy release rate reduces also with growth of parameters, $\beta_1$ and $\beta_5$, which characterize the change of $E$ across the thickness and length of the beam. Growth of $l/b$ and $h_Q/l$ ratios generates significant rise of the strain energy release rate. The non-linear character of the beam mechanical behavior under impact loading and temperature field also generates rise of the strain energy release rate (indication for this is the effect of the change of parameter, $\beta_2$).

## Recommendations

The approach developed in this paper can be recommended for analyzing the lengthwise facture in continuously inhomogeneous beam structures under impact loading and temperature field.

## Scientific Ethics Declaration

* The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

## Conflict of Interest

* The author declares that he has no conflicts of interest.

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