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Influence of Boundary Conditions of a Contact Problem on the Stress Distribution in a Semi-Infinite Space

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Abstract: The main tasks that arise in the process of mining are the creation of conditions that ensure the stability, strength and reliability of rock formations, allowing efficient and safe implementation of technological modes of mining. It should be noted that the annual damage caused by landslides around the world amounts to huge amounts commensurate with earthquake damage, the same ratio with human casualties. Therefore, the problem of quantitative forecasting of stability, creep and strength of slopes and slopes is of paramount national economic importance. The purpose of this work, carried out within AP19678682 grant project, is to develop a methodology for calculating the stress state of a half-space under the action of massive bodies in conditions of a rough contact surface. In the course of the conducted research, a mathematical model of the stress state of a half-space in conditions of a rough contact surface was developed. A comparative analysis of the study results of stress state at elastic half-space under the action of a massive body under conditions of a smooth and rough contact surface showed that the normal stresses with a smooth surface are 1 in the center and 1.4 on the sides, and the rough contact surface on the contact has stresses 1 in the center and 1.1 on the sides. The surface fades at a depth of 400, and with a rough surface it fades at a depth of 300. Analysis of the obtained results of the distribution of normal and tangential stresses in the depth of the array showed that the greater the width of the base.

Keywords: Elasticity theory, Function argument, Cauchy-Riemann relations, Laplace equations, Boundary conditions

Introduction

In soil mechanics, the general patterns of interaction under the load of rocks of different deformability, stability and strength are considered. To create a mathematical model of the stress-strain state of soils, different directions of continuum mechanics are used: theoretical mechanics, elasticity theory, plasticity theory and dynamic processes theory (Bartolomey, 2003). Preliminary analysis shows that rock loading occurs for different reasons and under different conditions of their interaction. There is a need to assess the stress state when creating artificial slopes, channels, dams and quarries. When implementing underground workings, great

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attention is paid to the stability of rocks and the peculiarities of the stress-strain state of array (Timoshenko & Goodyear, 1979). The main problems are caused by the complexity of building and maintaining mining and capital workings in tectonically stressed low-strength massifs of fractured rocky rocks. Under these conditions, despite the relatively high strength of rocks in the samples, disturbances in the stability of the contour array of underground workings occur even with relatively small outcrops and low stress and strain levels. In underground workings, the strength of the massif is affected by numerous chaotic cracks and differently oriented tectonic disturbances on the structural blocks. The probabilistic nature and spatiotemporal variability of these indicators necessitate conducting field-based instrumental studies at various scale levels.

The main tasks that arise in the process of mining are the creation of conditions that ensure the stability, strength and reliability of rock formations, allowing efficient and safe implementation of technological modes of mining (Chigirinsky & Putnoki 2017). It should be noted that the annual damage caused by landslides around the world amounts to huge amounts commensurate with earthquake damage, the same ratio with human casualties. Therefore, the problem of quantitative forecasting of stability, creep and strength of slopes and slopes is of paramount national economic importance. The objectives of the research carried out in this work are: to develop a mathematical model of the stress state of a half-space under conditions of a rough contact surface; to study the stress state of an elastic half-space under the action of a massive body under conditions of a rough contact surface; to analyze the obtained result of the distribution of normal and tangential stresses in the depth of the array.

Method

The study of the stress state in elastic and plastic half-spaces in mountain massifs of different depths is an urgent problem of continuum mechanics. At the present stage, the method of argument of a function by a complex variable is effectively used in solutions, however, it can be seen from the presented analyses that the influence of tangential stresses is not fully represented, which does not allow an adequate assessment of its effect on the strength characteristics of rocks. There is a need to strengthen well-known solutions at the current level of implemented solutions and ensure the real reliability of the result. For the planar problem, three equations of elasticity theory, two differential equations of equilibrium, the stress strain continuity condition, and boundary conditions are chosen:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}=0,\ \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0,\ \nabla^2(\sigma_x+\sigma_y)=\nabla^2(2\cdot\sigma_0)=0. \quad (1)$$

Boundary conditions in the stresses:

$$\tau_n=-\frac{\sigma_x-\sigma_y}{2}\cdot sin2\varphi+\tau_{xy}\cdot cos2\varphi. \quad (2)$$

In (Chigirinsky & Putnoki 2017), solutions to the plane problem of elasticity theory in Cartesian coordinates are presented. The analytical solution of this problem is presented in the form:

$$\sigma_x=C_\sigma exp(-\theta)\,cosA\Phi+\sigma_0+C,$$

$$\sigma_y=-C_\sigma exp(-\theta)\,cosA\Phi+\sigma_0+C, \quad (3)$$

$$\tau_{xy}=\exp(-\theta)\,C_\sigma sinA\Phi\sigma_0=\pm n\cdot C_\sigma\,exp(-\theta)\,cosA\Phi,$$

at $\theta_x=A\Phi_y,\ \theta_y=-A\Phi_x,\ A\Phi_{xx}+A\Phi_{yy}=0,\ \theta_{xx}+\theta_{yy}=0.$

where $\sigma_x, \sigma_y, \tau_{xy}$ – components of the stress tensor; σ_0 – average normal stress.

Taking into account the boundary conditions, expression (3) was presented as:

$$\sigma_y=2exp\left[\frac{1}{2}\cdot AA_6(x^2-y^2)\right]C_\sigma\cos(AA_6xy), \quad (4)$$

$$\tau_{xy}=exp\left[\frac{1}{2}\cdot AA_6(x^2-y^2)\right]C_\sigma sin(AA_6xy).$$

Exponent indicator θ and the argument of the trigonometric function AΦ were derived from the Laplace equations and the Cauchy-Riemann relations. In accordance with the task, they must meet the stress boundary conditions:

$$\sigma_y = 2exp\left[\frac{1}{2}\cdot\frac{\pi}{2b\,y_0}(x^2)\right]C_\sigma,\ \tau_{xy} = 0. \tag{5}$$

Results and Discussion

In accordance with formula (5), contact stresses and stresses in the depth of the array were calculated (Figure 1). From solution (4) it can be seen that zero tangential stresses do not negate their presence in the depth of the half-space. A stable attenuating function in the depth of the half-space and a concave plot of contact normal stresses are obtained.

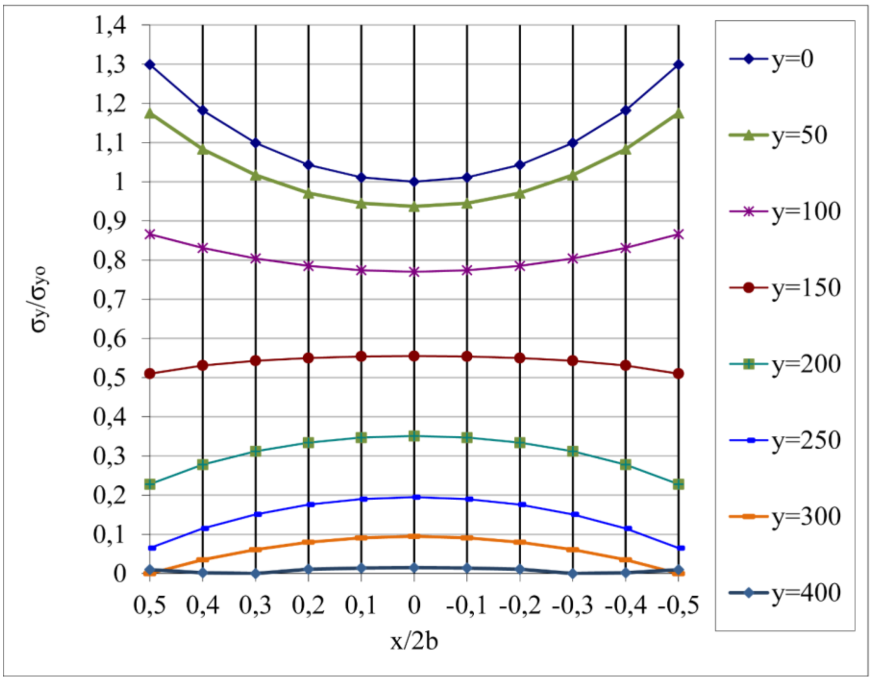

Figure 1. Distribution of normal stresses at the contact and in the depth of the half-space under the action of a flat die without regard to friction

With this formulation of the question, it is of theoretical and practical interest to solve the problem of a rough contact surface. Let's use the formulation of problem (1) and boundary conditions (2). Simplifying the boundary conditions through trigonometric substitution, the first argument function AΦ is introduced. From the solution condition, a second argument is introduced, the function θ, which determines the fundamental substitution of exp(-θ). Taking into account the trigonometric and fundamental substitution in the equation of continuity of deformation, taking into account the function of the complex variable [6], the differential equations are obtained in the form:

$$exp(\theta + iA\Phi)\cdot\left[\left(\theta_{xx}+\theta_{yy}\right)+\left(A\Phi_{xx}+A\Phi_{yy}\right)\cdot i+(\theta_x+iA\Phi_x)^2+\left(\theta_y+iA\Phi_y\right)^2\right]+$$
$$+exp(\theta - iA\Phi)\cdot\left[\left(\theta_{xx}+\theta_{yy}\right)-\left(A\Phi_{xx}+A\Phi_{yy}\right)\cdot i+(\theta_x-iA\Phi_x)^2+\left(\theta_y-iA\Phi_y\right)^2\right]=0. \tag{6}$$

The operators in formula (6) located near the exponent contain the same second derivatives in terms of coordinates and non-linearity. If, for some reason, the operators are zero, then the identity holds. Let's write down the nonlinearities in the operators and rearrange them.

$$(\theta_x + iA\Phi_x)^2 + (\theta_y + iA\Phi_y)^2 = (\theta_x + A\Phi_y)\cdot(\theta_x - A\Phi_y) + 2i(\theta_x \cdot A\Phi_x + \theta_y \cdot A\Phi_y) + \\ +(\theta_y + A\Phi_x)\cdot(\theta_y - A\Phi_x),$$

$$(\theta_x - iA\Phi_x)^2 + (\theta_y - iA\Phi_y)^2 = (\theta_x + A\Phi_y)\cdot(\theta_x - A\Phi_y) - \\ -2i(\theta_x \cdot A\Phi_x + \theta_y \cdot A\Phi_y) + (\theta_y + A\Phi_x)\cdot(\theta_y - A\Phi_x).$$

It follows that this differential equation will be satisfied when the Cauchy-Riemann relations and the Laplace equation are fulfilled:

$$\theta_x = \mathrm{A}\Phi_y, \quad \theta_y = -\mathrm{A}\Phi_x,$$

$$\mathrm{A}\Phi_{xx} + \mathrm{A}\Phi_{yy} = 0, \tag{7}$$

$$\theta_{xx} + \theta_{yy} = 0.$$

It is possible to obtain a new solution in the interaction of bodies with a rough contact surface. As a result of solving differential equations (7), we have:

$$\mathrm{A}\Phi = AA_6 x(y + C), \tag{8}$$

The function (8) satisfies the Laplace equation, i.e. we have: $\mathrm{A}\Phi_{xx} = \mathrm{A}\Phi_{yy} = 0.$

Thus, equation (8) defines new boundary conditions that will be related to the roughness of the contact surface. Using the Cauchy-Riemann relations, the second argument function θ is determined. The exponent indicator θ is written as:

$$\theta = AA_6 \frac{x^2 - (y+C)^2}{2}. \tag{9}$$

Taking into account expressions (8) and (9), the normal and tangential stress takes the form:

$$\sigma_y = 2exp\left[AA_6 \frac{x^2-(y+C)^2}{2}\right] C_\sigma cos[AA_6 x(y + C)],$$

$$\tau_{xy} = exp\left[AA_6 \frac{x^2-(y+C)^2}{2}\right] C_\sigma sin[AA_6 x(y + C)]. \tag{10}$$

Based on the analysis of the expressions obtained (10), it is established that:

$$C = f \cdot b,$$

where *f* and *b* - the friction coefficient on the contact surface and the half-width of the massive base.

Boundary conditions: at $x= b$, $y= 0$, $\sigma_y= k_1$, $\tau_{xy}=f \cdot k_1$, $A\Phi= A\Phi_1$, $\theta= \theta_1$. Substituting the boundary conditions in solution (4), we find the constant AA_6:

$$AA_6 = \frac{f}{b \cdot C} = \frac{f}{b \cdot f \cdot b} = \frac{1}{b^2}.$$

As a result, a mathematical model of stress state of a half-space under conditions of a rough contact surface has been developed. Based on expressions (10), studies of the stress state of the array under the action of massive external bodies with a rough contact surface were carried out. Figures 2 and 3 show the distributions of contact normal and tangential stresses into the depth of the array, taking into account the influence of the width of the base, as well as the friction coefficient. Comparing the results of the study with the data of other authors, we are convinced that they are qualitatively and quantitatively the same. At the contact with the stamp in a semi-infinite space, the normal contact stress diagram has a concave character. This indicates the reliability of the result obtained. In the depth of space, the stress state of the medium attenuates to zero.

As the analysis shows, under the influence of maximum tangential stresses, sliding lines develop in massifs, which are dangerous because they are sources of shear, collapse and subsidence of rock massifs. The distributions of tangential stresses are visible, the values of which are maximum at a depth of 70, in the angular loading zones such a stressed soil condition shows the possibility of destruction under the influence of tangential stresses, taking into account the friction coefficient.

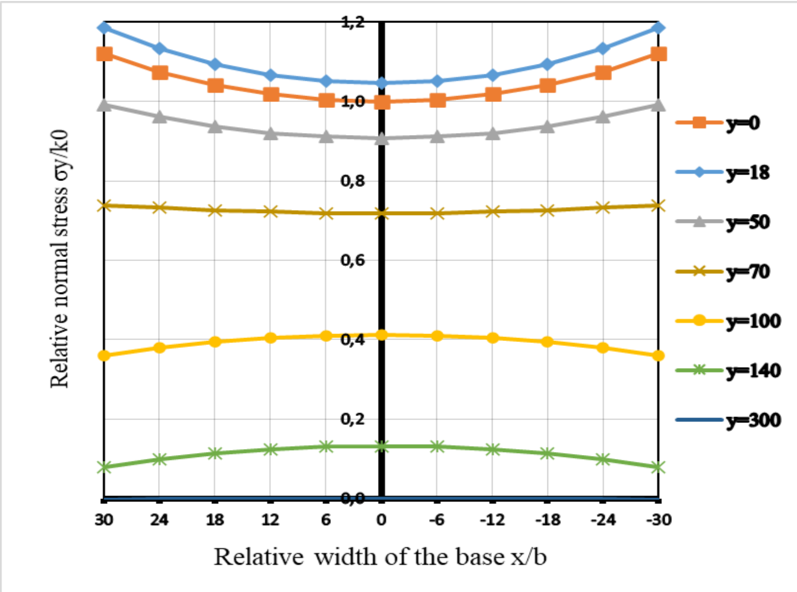

Figure 2. Distribution of normal stress at the contact and in the depth of the array with a coefficient of friction f=0.3 and a width b=60

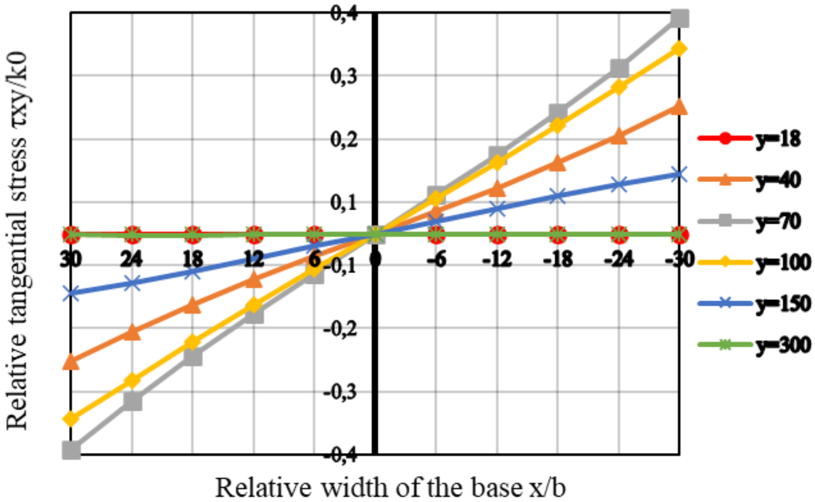

Figure 3. Distribution of tangential stress at the contact and in the depth of an array with a coefficient of friction f=0.3 and a width b=60

Figure 4 shows the effect of the friction coefficient f = 0.1-0.5 on the distribution of normal stresses in depth along the edges of the die x = b and in the center x = 0, with a base width of b = 60. Figure 5 shows the effect of the coefficient of friction f = 0.1-0.5 on the distribution of tangential stresses in depth along the edges of the die x = b, with a base width of b = 60. Figure 6 shows the effect of the base width b = 20-100 on the distribution of normal stresses in depth along the center x = 0, with a friction coefficient f = 0.3.

It can be seen from the obtained graphs (Figure 4) that with an increase in the friction coefficient, the attenuation depth of normal stresses does not increase significantly. Figure 1 shows that the metal flows from a high-load zone to a lower-load zone. Judging by the drawing, the metal spreads out from the center in the horizontal axis in the opposite direction, which gives a tangential stress of one sign. The tangential stresses in the contact layers move to the center. This position is explained by a change in the sign on the contact (zone of obstructed deformations, Figure 5). As the friction coefficient increases, the depth of the maximum tangential stress increases, and the attenuation depth is identical to the normal stress. It is also shown (Figure 6) that with increasing base width; the attenuation depth of normal stresses increases significantly.

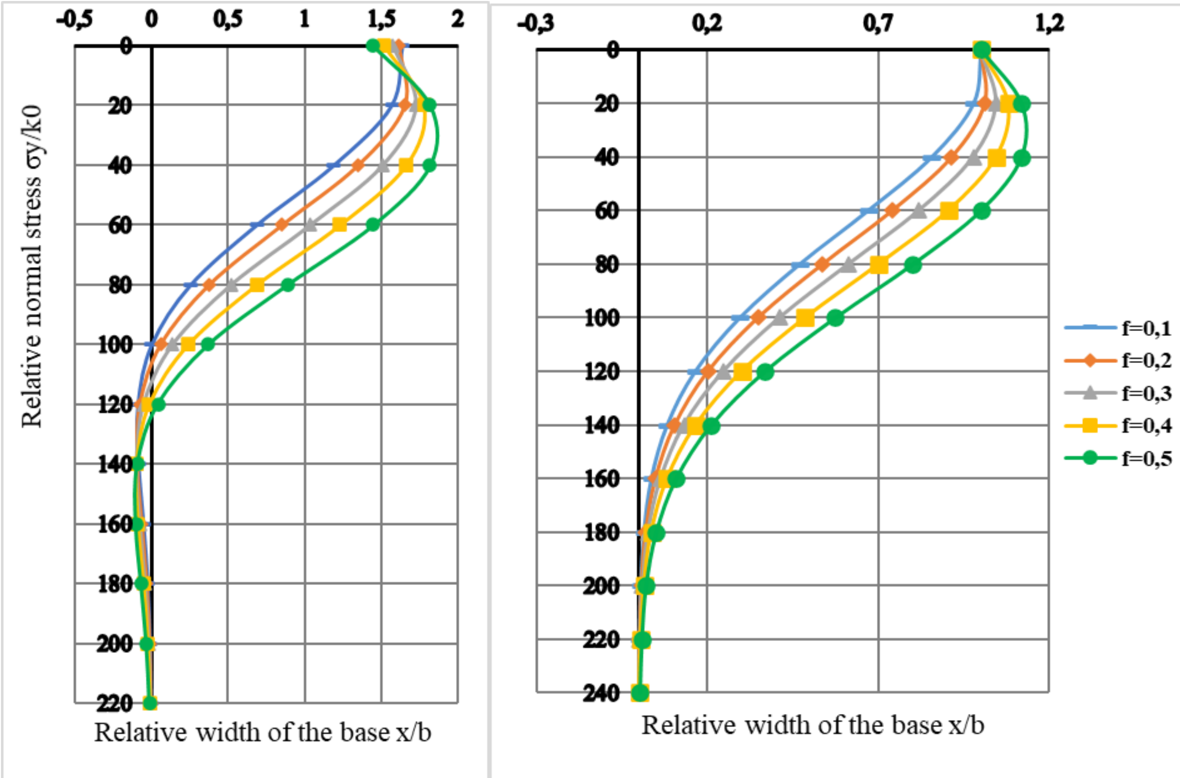

Figure 4. Distribution of normal stresses at the contact and in the depth of the array along the edges of the die and in the center with a base width of b =60

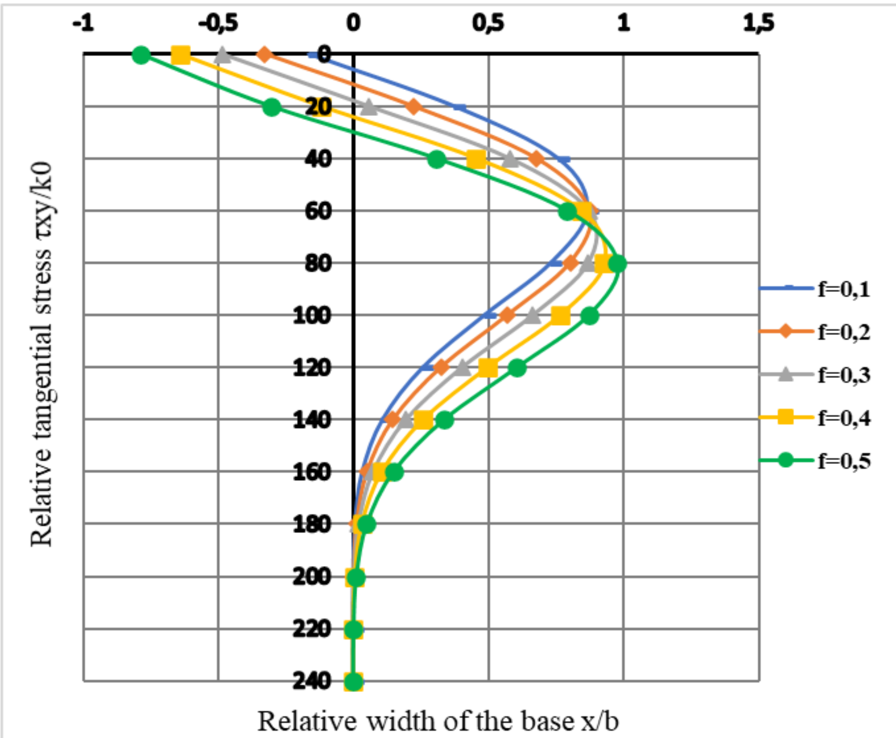

Figure 5. Distribution of tangential stresses at the contact and in the depth of the array along the edges of the die with a base width of b =60

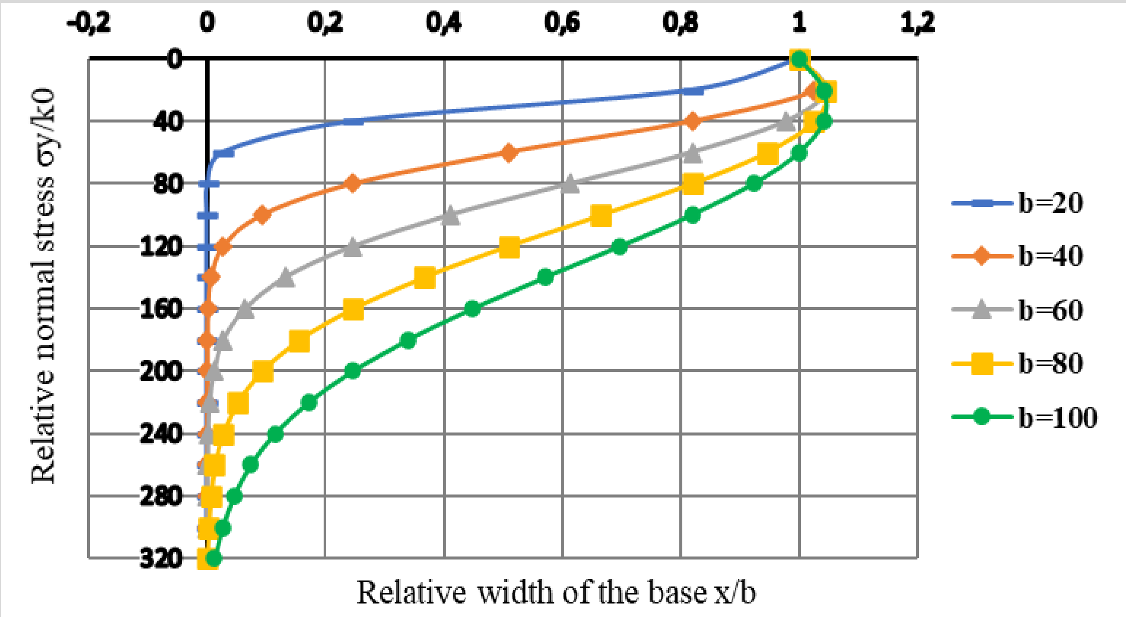

Figure 6. Distribution of normal stresses at the contact and in the depth of the array in the center of the die, with different widths with a friction coefficient f=0.3

Conclusion

In the course of the conducted research, a mathematical model of the stress state of a half-space in conditions of a rough contact surface was developed. A comparative analysis of the results of the study of the stress state of an elastic half-space under the action of a massive body under conditions of a smooth and rough contact surface showed that normal stresses with a smooth surface in the center are 1 and on the sides 1.4, and the rough contact surface on the contact has stresses 1 in the center and 1.1 on the sides. In this case, normal stresses with a smooth surface fade at a depth of 400, and with a rough surface they fade at a depth of 300. Analysis of the obtained results of the distribution of normal and tangential stresses in the depth of the array showed that the greater the width of the base, the greater the depth of the normal stresses attenuate.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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References

Bartolomey, A. A. (2003). *Mechanics of soils: A textbook for students*. ACB.

Chigirinsky, V., & Putnoki, A. (2017). Development of a dynamic model of transient processes in mechanical systems using argumentative functions. *Eastern European Journal of Corporate Technologies, 3/7*(87), 11-21.

Timoshenko, S. P., & Goodyear, J. (1979). *Elasticity theory*. Main editorial office of physical-mathematical literature.

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