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Nano Ideal Semi Alpha Generalized Continuous and Irresolute Mappings in Nano Ideal Topological Spaces

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Abstract: Topology, though relatively recent in its development, has found remarkable applications across nearly all fields. While the theoretical or fundamental aspects of topology may seem abstract or dry at first, its applied side becomes truly fascinating once one becomes familiar with it. This research explores a variety of applications in diverse areas of Science and Technology-including Biology, Robotics, Geographic Information Systems (GIS), Engineering, Computer Science, and the Medical field. Though topology originates from mathematics, its influence extends far beyond, making a profound impact on numerous disciplines with its powerful and wide-ranging applications. The purpose of this paper is to present another class of functions referred to as Nano ideal semi α-generalized continuous maps, NIsαg-irresolute maps, NIsαg-open maps, NIsαg-closed maps, strongly NIsαg-open maps, strongly NIsαg-closed maps, NIsαg-homeomorphisms, Nano quasi Isαg-open maps, and Nano quasi Isαg-closed maps in nano ideal topological spaces. We investigate several properties and characterizations of this class of mappings in nano-ideal topological spaces.

Keywords: Nano ideal topological space, Nisαg-open set, Nisαg-closed set, Nisαg-continuous map, Nisαg-irresolute map, Nisαg-open map, Nisαg-closed map, Strongly nisαg-open map, Strongly nisαg-closed map, Nisαg-homeomorphism, Nqisαg-open map, Nqisαg-closed map

Introduction

Nano Ideal Topological Spaces, an extension of classical nano topology, have proven valuable in a wide range of domains where data is incomplete, uncertain, or imprecise. Their applications span various complex systems, including network anomaly detection, social and biological network analysis, transportation network optimization, pattern recognition, IoT security, financial market analysis, environmental monitoring, healthcare systems analysis, and supply chain management.

Nano ideal topological spaces generalize classical topological spaces by integrating "ideals"-collections of subsets with specific structural properties-and introducing various forms of "nano" sets and mappings. These structures allow for the analysis of relationships between points and subsets at an extremely fine-grained level, analogous to the "nano" concept in other scientific disciplines. Research in this area focuses on defining and studying different types of nano open and closed sets, as well as examining the behaviour of various nano mappings, such as nano-continuous, nano-irresolute, and nano-open maps. The aim is to understand their properties, explore their interconnections, and identify possible applications in theoretical mathematics and potentially in other fields.

The concept of continuity is fundamental in a large part of contemporary mathematics. In the nineteenth century, definitions of continuity were formulated for functions of real or complex variables. In the early twentieth century, the concept of continuity was generalized to apply to functions between topological spaces. Over the years, many generalizations of continuous mapping have been introduced and studied. Levine (1963), Mashhour et al.(1982) and Mashhour et al. (1983) have introduced the notions of semi-continuity, pre-continuity and α-continuity, respectively. Continuous and irresolute functions in topological spaces present a new method in the direction of research. Irresolute maps are introduced and studied by associated sciences. Parimala et al. (2018) presented the concept of nano-ideal generalized closed sets in nano-ideal topological spaces and investigated a portion of their essential properties. Recently, Dr. J. Arul Jesti and J. Joycy Renuka (2022) presented and concentrated on nIgsemi∗-closed sets and nIgsemi∗-open sets, characterizing nIgsemi∗-continuous and nIgsemi∗-irresolute functions in nano ideal topological spaces.

In 2021, Suganya et al. defined and presented a new class of sets called Nano ideal NIsαg-closed sets and NIsαg-open sets in Nano ideal topological spaces. In this paper, the conceptualization of NIsαg-open sets and NIsαg-closed sets in nano ideal topological spaces is utilized to characterize and examine another class of mappings called NIsαg-continuous maps, NIsαg-irresolute maps, NIsαg-open maps, NIsαg-closed maps, strongly NIsαg-open maps, strongly NIsαg-closed maps, NIsαg-homeomorphisms, Nano quasi Isαg-open maps, and Nano quasi Isαg-closed maps in nano ideal topological spaces. All these concepts will be helpful for further generalizations of nano-continuous and nano-irresolute mappings in nano-ideal topological spaces. We have investigated fundamental properties and characterizations of this class of mappings in nano ideal topological spaces.

Preliminaries

Throughout this paper $\left(U,T_R(X)\right)$ is a Nano topological space $\left(\text{briefly, } \mathcal{N}_{ano}-\text{Top}-\text{Space}\right)$ and $\left(U,T_R(X),I\right)$ is a Nano ideal topological space $\left(\text{briefly, } \mathcal{N}_{ano}-\text{Ideal}-\text{Top}-\text{Space}\right)$ concerning X where $X \subseteq U$, R is an equivalence relation on U/R denotes the family of equivalence classes of U by R. Here, we recall the following known definitions and properties.

Definition 2.1. Let U be a non-empty finite set of objects called the universe, and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be discernible from one another. The pair (U,R) is said to be the approximation space. Let $X \subseteq U$.

1. The Lower approximation of the set X with respect to the relation R is the set of all objects that can be for certain, classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X)=\bigcup_{x\in U}\{R(x): R(x)\subseteq X\}$ where $R(x)$ denotes the equivalence class determined by X.
2. The upper approximation of X concerning R is the set of all objects that can be possibly defined as X concerning R and it is denoted by $U_R(X)$. That is $U_R(X)=\bigcup_{x\in U}\{R(x): R(x)\cap X\neq\phi\}$.
3. The boundary region of X with respect to R is the set of all objects that can be classified neither in nor as not X with respect to R and is denoted by $B_R(X)$. That is, $B_R(X)=U_R(X)-L_R(X)$.

Definition 2.2. Let U be a non-empty finite set of objects called the universe and R be an the universe, R be an equivalence relation on U named as the and $T_R(X)=\{U,\phi,L_R(X),U_R(X),B_R(X)\}$ where $X\subseteq U$. Then $T_R(X)$ satisfies the following axioms.

(1) U and $\phi\in T_R(X)$.

(2) The union of any subcollection of $T_R(X)$ is in $T_R(X)$.

(3) The intersection of the elements of any finite subcollection of $T_R(X)$ is in $T_R(X)$.

Then $T_R(X)$ is a topology on U called the Nano topology on U concerning X, the pair $\left(U,T_R(X)\right)$ is called the Nano topological space $\left(\text{Briefly } \mathcal{N}_{ano}-\text{Top}-\text{Space}\right)$. The elements of $T_R(X)$ are called nano-open $\left(\mathcal{N}_{ano}-\text{open}\right)$ sets. The complements of the nano-open sets are called nano-closed $\left(\mathcal{N}_{ano}-\text{closed}\right)$ sets.

Definition 2.3. Let $\left(U,T_R(X)\right)$ be a $\mathcal{N}_{ano}-\text{Top}-\text{Space}$ and $A\subseteq X$. Then

*. The Nano interior of a set A is defined as the union of all $\mathcal{N}_{ano}-\text{open}$ subsets contained in A and it is denoted by $\mathcal{N}_{ano}-\text{Int}(A)$. $\mathcal{N}_{ano}-\text{Int}(A)$ is the largest $\mathcal{N}_{ano}-\text{open}$ subset of A.

**. The Nano closure of a set A is defined as the intersection of all $\mathcal{N}_{ano}$ −closed sets containing A and it is denoted by $\mathcal{N}_{ano}-Cl(A)$. $\mathcal{N}_{ano}-Cl(A)$ is the smallest $\mathcal{N}_{ano}$ −closed set containing A.

Definition 2.3. An ideal I on a $Top-Space$ is a non-empty collection of subsets of X which satisfies (1) $A\in I$ and $B\subseteq A\Rightarrow B\in I$. (2) $A\in I$ and $B\in I\Rightarrow A\bigcup B\in I$.

Definition 2.4. A nano topological space $(U,T_R(X))$ with an ideal I on U is called a nano ideal topological space or nano ideal space and denoted as $(U,T_R(X),I)$.

Definition 2.5. Let $(U,T_R(X),I)$ be a nano ideal topological space. A set operator $(A)^{*N}:P(U)\to P(U)$ is called the nano local function of I on U with respect to I on $T_R(X)$ is defined as $(A)^{*N}=\{x\in U: G\bigcap A\notin I, \text{for each } x\in G, G\in T_R(X)\}$. and is denoted by $(A)^{*N}$, where nano closure operator is defined as $N_{ano}Cl^*(A)=A\bigcup(A)^{*N}$.

Prosition 2.6. Let $(U,T_R(X),I)$ be a nano ideal topological space and let A and B be subsets of U, then

(1). $(\phi)^{*N}=\phi$

(2). $A\subseteq B\Rightarrow(A)^{*N}\subseteq(B)^{*N}$.

(3). $I\subseteq J\Rightarrow(A)^{*N}(J)\subseteq(A)^{*N}(I)$.

(4). $(A)^{*N}\subseteq N_{ano}Cl^*(A)$.

(5). $(A)^{*N}$ is a N_{ano} $-closed\ set$.

(6). $((A)^{*N})^{*N}\subseteq(A)^{*N}$.

(7). $(A)^{*N}\bigcup(B)^{*N}=(A\bigcup B)^{*N}$.

(8). $(A)^{*N}\bigcap(B)^{*N}=(A\bigcap B)^{*N}$.

(9). *For every* N_{ano} *−open set* G, $G\bigcap(G\bigcap A)^{*N}\subseteq(G\bigcap A)^{*N}$.

(10). *For every* $K\in I$, $(A\bigcup K)^{*N}=(A)^{*N}=(A-K)^{*N}$.

Remark 2.6. Let $(U,T_R(X),I)$ be a $\mathcal{N}_{ano}-Ideal-Top-Space$ and A be a subset of U, if $A\subseteq(A)^{*N}$, then $(A)^{*N}=N_{ano}Cl(A)^{*N}=N_{ano}Cl(A)=N_{ano}Cl^*(A)$.

Definition 2.7. Let $(U,T_R(X),I)$ be a $\mathcal{N}_{ano}-Ideal-Top-Space$ and $A\subseteq U$. Then A is called

(1) $\mathcal{N}_{ano}-semi-open$ set if $A\subseteq\mathcal{N}_{ano}-Cl[\mathcal{N}_{ano}-Int(A)]$.

(2) $\mathcal{N}_{ano}-semi-closed$ set if $\mathcal{N}_{ano}-Int[\mathcal{N}_{ano}-Cl(A)]\subseteq A$.

(3) $\mathcal{N}_{ano}\alpha-closed$ set if $\mathcal{N}_{ano}-Cl[\mathcal{N}_{ano}-Int(\mathcal{N}_{ano}-Cl(A))]\subseteq A$.

(4) $\mathcal{N}_{ano}\alpha-open$ set if $A\subseteq\mathcal{N}_{ano}-Int[\mathcal{N}_{ano}-Cl(\mathcal{N}_{ano}-Int(A))]$.

(5) $\mathcal{N}_{ano}-generalized$ closed (briefly $\mathcal{N}_{ano}g-closed$) set if $\mathcal{N}_{ano}-Cl(A)\subseteq G$ whenever $A\subseteq G$ and G is $\mathcal{N}_{ano}-open$ in U.

The complements of the above-mentioned sets are called their respective open sets.

Definition 2.8. Let $(U,T_R(X),I)$ be a $\mathcal{N}_{ano}-Ideal-Top-Space$ and $A\subseteq U$. Then

(1) $\mathcal{N}_{ano}*-closed$, set if $(A)^{*N}\subseteq A$.

(2) $\mathcal{N}_{ano}*-dense$ set if $A\subseteq(A)^{*N}$.

(3) $\mathcal{N}_{ano}$ Ig–closed if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is $\mathcal{N}_{ano}$ –open in U.

(4) $\mathcal{N}_{ano}$ Ig⋆–closed if $(A)^{*N} \subseteq G$ whenever $A \subseteq G$ and G is $\mathcal{N}_{ano}$g–open in U.

Definition 2.9. Let $(U, T_R(X), I)$ be a $\mathcal{N}_{ano}$ *–Ideal –Top –Space* and $A \subseteq U$. Then A is $\mathcal{N}_{ano}$ *–dense in itself (resp. $\mathcal{N}_{ano}$ *–perfect and $\mathcal{N}_{ano}$ *–closed set) if $A \subseteq (A)^{*N}$ (resp. $A = (A)^{*N}, (A)^{*N} \subseteq A$).

Lemma 2.10. Let $(U, T_R(X), I)$ be a $\mathcal{N}_{ano}$ *–Ideal –Top –Space* and $H \subseteq U$. If H **is** $\mathcal{N}_{ano}$ *–dense in itself, then $(H)^{*N} = N_{ano}Cl(H)^{*N} = N_{ano}Cl(H) = N_{ano}Cl^*\left[(H)^{*N}\right]$.

Definition 2.11. Let $(U, T_R(X), I)$ be a $\mathcal{N}_{ano}$ *–Ideal –Top –Space* and $A \subseteq U$. Then A is said to be $\mathcal{N}_{ano}S\alpha$–open set if there exists a $\mathcal{N}_{ano}S$–open set P in U such that $P \subseteq H \subseteq \mathcal{N}_{ano}$–$Cl(P)$ or equivalently if $H \subseteq \mathcal{N}_{ano}$–$Cl(\mathcal{N}_{ano}\alpha$–$Int(P))$.

Result 2.12. Let $(U, T_R(X))$ be a $\mathcal{N}_{ano}$ *–Top –Space.* Then

(i) .Every $\mathcal{N}_{ano}$ –open set is $\mathcal{N}_{ano}S\alpha$–open set.

(ii) .Every $\mathcal{N}_{ano}\alpha$–open set is $\mathcal{N}_{ano}S\alpha$–open set.

Definition 2.13. Let $(U, T_R(X))$ be a $\mathcal{N}_{ano}$ *–Top –Space* and $H \subseteq U$. Then H is said to be a Nano semi α-generalized (briefly, $\mathcal{N}_{ano}S\alpha g$–closed) set if if $\mathcal{N}_{ano}$–$Cl(H) \subseteq G$ whenever $H \subseteq G$ and G is $\mathcal{N}_{ano}S\alpha$–open set in U. The complement $H^C = U - H$ of a $\mathcal{N}_{ano}S\alpha$–closed set H is called a $\mathcal{N}_{ano}S\alpha$–open in U.

Definition 2.14. Let $(U, T_R(X), I)$ be a $\mathcal{N}_{ano}$ *–Ideal –Top –Space* and $H \subseteq U$. Then H is said to be a Nano ideal semi α-generalized (briefly, $\mathcal{N}_{ano}IS\alpha g$–closed) set if if $(H)^{*N} \subseteq G$ whenever $H \subseteq G$ and G is $\mathcal{N}_{ano}S\alpha$–open set in U. The complement $H^C = U - H$ of a $\mathcal{N}_{ano}IS\alpha g$–closed set H is called a $\mathcal{N}_{ano}IS\alpha g$–open set in U. The collection of all $\mathcal{N}_{ano}IS\alpha g$–open (resp. $\mathcal{N}_{ano}IS\alpha g$–closed) sets is denoted by $\mathcal{N}_{ano}IS\alpha g$–$O(U, T_R(X), I)$ (resp. $\mathcal{N}_{ano}IS\alpha g$–$O(U, T_R(X), I)$).

Definition 2.15. Let $(U, T_R(X), I)$ be a $\mathcal{N}_{ano}$ *–Ideal –Top –Space* and $H \subseteq U$. Then

*. The $\mathcal{N}_{ano}IS\alpha$ – interior of a set A is defined as the union of all $\mathcal{N}_{ano}IS\alpha g$–open subsets contained in A and it is denoted by $\mathcal{N}_{ano}IS\alpha g$–$Int(A)$. $\mathcal{N}_{ano}IS\alpha g$–$Int(A)$ is the largest $\mathcal{N}_{ano}IS\alpha g$–open subset of A.

**. The $\mathcal{N}_{ano}IS\alpha g$–closure of a set A is defined as the intersection of all $\mathcal{N}_{ano}IS\alpha g$–closed sets containing A and it is denoted by $\mathcal{N}_{ano}IS\alpha g$–$Cl(A)$. $\mathcal{N}_{ano}IS\alpha g$–$Cl(A)$ is the smallest $\mathcal{N}_{ano}IS\alpha g$–closed set containing A.

Remark 2.16. $\mathcal{N}_{ano}$ –open $\Rightarrow$ $\mathcal{N}_{ano}$ ⋆–open $\Rightarrow$ $\mathcal{N}_{ano}IS\alpha g$–open $\Rightarrow$ $\mathcal{N}_{ano}Ig$–open.

& $\mathcal{N}_{ano}$ –closed $\Rightarrow$ $\mathcal{N}_{ano}$ ⋆–closed $\Rightarrow$ $\mathcal{N}_{ano}IS\alpha g$–closed $\Rightarrow$ $\mathcal{N}_{ano}Ig$–closed.

3 Nano Ideal Semi –α –Generalized –Continuous Mappings

Definition 3.1. A map $f : (U, T_R(X), I) \to (V, \sigma_{R'}(Y), J)$ is $\mathcal{N}_{ano}$–continuous if $f^{-1}(H)$ is a $\mathcal{N}_{ano}$–closed for every $\mathcal{N}_{ano}$–closed set H of $(V, \sigma_{R'}(Y), J)$.

Proposition 3.2. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ −continuous if and only if $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{ano}$ −open set for every $\mathcal{N}_{ano}$ −open set H of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Definition 3.3. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ *−continuous if $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{ano}$ *−closed for every $\mathcal{N}_{ano}$ −closed set H of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Proposition 3.4. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ *−continuous if and only if $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{ano}$ *−open for every $\mathcal{N}_{ano}$ −open set H of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Definition 3.5. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous if and only if $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−closed set for every $\mathcal{N}_{ano}$ −closed set H of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Theorem 3.6. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous if and only if $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ for every $\mathcal{N}_{ano}$ −open H In $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Proof. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous and H be a $\mathcal{N}_{ano}$ −open set in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Then H^{C} is $\mathcal{N}_{ano}$ −closed in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ and since f is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous, $f^{-1}(\mathrm{H}^{C})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−closed in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. But $f^{-1}(\mathrm{H}^{C})=(f^{-1}(\mathrm{H}))^{C}$ and so $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$.

Conversely, assume that $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ for each $\mathcal{N}_{ano}$ −open set H in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Let F be a $\mathcal{N}_{ano}$ −closed set in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Then F^{C} is $\mathcal{N}_{ano}$ −open in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ and by assumption, $f^{-1}(\mathrm{F}^{C})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Since $f^{-1}(\mathrm{F}^{C})=(f^{-1}(\mathrm{F}))^{C}$, we have $f^{-1}(\mathrm{F})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−closed in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ and so f is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous.

Theorem 3.7 Every $\mathcal{N}_{ano}$ −continuous function $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous but not conversely.

Proof. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be a $\mathcal{N}_{ano}$ −continuous function. Therefore, for each $\mathcal{N}_{ano}$ −open set H of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{ano}$ −open set in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Since $\mathrm{T}_{\mathrm{R}}(\mathrm{X})\subseteq\mathcal{N}_{ano}\mathrm{IS}\alpha g\text{−}O(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. So $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open set and so f is a $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous function.

Proposition 3.8. Every $\mathcal{N}_{ano}$ *−continuous function is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−continuous. but not conversely.

Proof. The proof is like Theorem 3.7, as every $\mathcal{N}_{ano}$ *−open(resp. $\mathcal{N}_{ano}$ *−closed) set is $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−open(resp. $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−closed)

Example 3.9. Let $\mathrm{U}=\{1,2,3,4\}$ with $\mathrm{U}/R=\{\{1,3\},\{2\},\{4\}\}$; $\mathrm{X}=\{2,3\}$ and $\mathrm{I}=\{\phi,\{3\}\}$. Then the $\mathcal{N}_{ano}$ −Top. $\mathrm{T}_{\mathrm{R}}(\mathrm{X})=\{\phi,\mathrm{U},\{2\},\{1,2,3\},\{1,3\}\}$. $\mathcal{N}_{ano}\mathrm{IS}\alpha g$−closed sets are $\phi,\mathrm{U},\{3\},\{4\},\{1,4\},\{2,4\},\{3,4\},\{1,2,4\},\{1,3,4\},\{2,3,4\}$. The set $\{1,4\}$ is not $\mathcal{N}_{ano}$ *−closed set.

Let $V=\{a,b,c\}$ with $V/R'=\{\{a\},\{b,c\}\}$ and $Y=\{a\}$. Then the $\mathcal{N}_{ano}$ –Top. $\Gamma_{R'}(Y)=\{\phi,\{a\},V\}$ and $J=\{\phi\}$. Define a function $f:(U,T_R(X),I)\to(V,\Gamma_{R'}(Y),J)$ by: $f(1)=b, f(2)=f(3)=a, f(4)=c$. Then $f^{-1}(\{a\})=\{2,3\}$ is $\mathcal{N}_{ano}$ ISαg–open but not $\mathcal{N}_{ano}$ *-open in U. Thus f is $\mathcal{N}_{ano}$ ISαg–continuous but not $\mathcal{N}_{ano}$ *-continuous. Furthermore, we notice that f is not $\mathcal{N}_{ano}$ –continuous, since $f^{-1}(\{a\})=\{2,3\}$ is not $\mathcal{N}_{ano}$ –open in $(U,T_R(X),I)$.

Theorem 3.10. Suppose that $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),K)$ are any two maps. Then gof is $\mathcal{N}_{ano}$ ISαg–continuous if g is $\mathcal{N}_{ano}$ *-continuous and f is $\mathcal{N}_{ano}$ ISαg–continuous.

Proof. Easy.

Definition 3.11. Let $(U,T_R(X),I)$ be a $\mathcal{N}_{ano}$ –ideal –Top. –Space. Let z be a point of U and G be a subset of U. Then G is called a $\mathcal{N}_{ano}$ ISαg–neighbourhood (briefly, $\mathcal{N}_{ano}$ ISαg–nbhd) of z in U if there exists a $\mathcal{N}_{ano}$ ISαg–open set S of U such that $z\in S\subseteq G$.

Proposition 3.12. Let M be a subset of $(U,T_R(X),I)$. Then $m\in\mathcal{N}_{ano}$ ISαg–Cl(M) if and only if for any $\mathcal{N}_{ano}$ ISαg–nbhd G_m of m in $(U,T_R(X),I)$, $M\cap G_m\neq\phi$.

Proof. Necessity. Assume $m\in\mathcal{N}_{ano}$ ISαg–Cl(M). Suppose that there is a $\mathcal{N}_{ano}$ ISαg–nbhd G of the point m in $(U,T_R(X),I)$ such that $G\cap M=\phi$. Since G is $\mathcal{N}_{ano}$ ISαg–nbhd of m in $(U,T_R(X),I)$, By definition, there exists a $\mathcal{N}_{ano}$ ISαg–open set H_m such that $m\in H_m\subseteq G$. Therefore, we have $H_m\cap M=\phi$ and so $M\subseteq(H_m)^C$. Since $(H_m)^C$ is an $\mathcal{N}_{ano}$ ISαg–closed set containing M, we have by definition $\mathcal{N}_{ano}$ ISαg–Cl$(M)\subseteq(H_m)^C$ and therefore $m\notin\mathcal{N}_{ano}$ ISαg–Cl(M), which is a contradiction.

Sufficiency. Assume for each $\mathcal{N}_{ano}$ ISαg–nbhd H_m of m in $(U,T_R(X),I)$, $H_m\cap M\neq\phi$. Suppose that $m\notin\mathcal{N}_{ano}$ ISαg–$Cl(M)$. Then by definition, there exists an $\mathcal{N}_{ano}$ ISαg–closed set F of $(U,T_R(X),I)$, such that $M\subseteq F$ and $m\notin F$. Thus $m\in F^C$ and F^C is $\mathcal{N}_{ano}$ ISαg–open in $(U,T_R(X),I)$, and hence F^C is a $\mathcal{N}_{ano}$ ISαg–nbhd of m in $(U,T_R(X),I)$. But $M\cap F^C=\phi$,, which is a contradiction.

In the next theorem we explore certain characterizations of $\mathcal{N}_{ano}$ ISαg–continuous maps.

Theorem 3.13. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ be a map. Then the following statements are equivalent.

(1). The map f is $\mathcal{N}_{ano}$ ISαg–continuous.

(2). The inverse of each $\mathcal{N}_{ano}$ –open set is $\mathcal{N}_{ano}$ ISαg–open.

(3). For each point p in $(U,T_R(X),I)$ and each $\mathcal{N}_{ano}$ –open set H in $(V,\sigma_{R'}(Y),J)$ with $f(p)\in H$, there is an $\mathcal{N}_{ano}$ ISαg–open set G in $(U,T_R(X),I)$ such that $p\in G, f(G)\subseteq H$.

(4). The inverse of each $\mathcal{N}_{ano}$ –closed set is $\mathcal{N}_{ano}$ ISαg–closed..

(5). For each p in $(U,T_R(X),I)$, the inverse of every neighbourhood of $f(p)$ is a $\mathcal{N}_{ano}$ ISαg–nbhd of p.

(6). For each p in $(U, T_R(X), I)$ and each neighbourhood M of $f(p)$, there is a $\mathcal{N}_{ano}\, IS\alpha g$–nbhd G of p such that $f(G) \subseteq M$.

(7). For each subset A of $(U, T_R(X), I)$, $f(\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(A)) \subseteq \mathcal{N}_{ano}\text{-}Cl(f(A))$.

(8). For each subset B of $(V, \sigma_{R'}(Y), J)$, $\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(B)))$.

Proof. $(1) \Leftrightarrow (2)$. This follows from Theorem 3.5.

$(2) \Leftrightarrow (4)$. The result follows from the fact that if Q is a subset of $(V, \sigma_{R^*}(Y), J)$, then $f^{-1}(Q^C) = (f^{-1}(Q))^C$.

$(2) \Leftrightarrow (5)$. For p in $(U, T_R(X), I)$, let M be a neighbourhood of $f(p)$. Then there exists an $\mathcal{N}_{ano}$–open set H in $(V, \sigma_{R'}(Y), J)$ such that $f(p) \in H \subseteq M$. Consequently, $f^{-1}(H)$ is an $\mathcal{N}_{ano}\, IS\alpha g$–open set in $(U, T_R(X), I)$ and $p \in f^{-1}(H) \subseteq f^{-1}(M)$. Thus $f^{-1}(M)$ is a $\mathcal{N}_{ano}\, IS\alpha g$–nbhd of p.

$(5) \Leftrightarrow (6)$. Let $p \in U$ and let M be a neighbourhood of $f(p)$. Then by assumption, $G = f^{-1}(M)$ is a $\mathcal{N}_{ano}\, IS\alpha g$–nbhd of p and $f(G) = f(f^{-1}(M)) \subseteq M$.

$(6) \Leftrightarrow (3)$. For an element p in $(U, T_R(X), I)$, let M be a $\mathcal{N}_{ano}$–open set containing $f(p)$. Then M is a neighbourhood of $f(p)$. So by assumption, there exists a $\mathcal{N}_{ano}\, IS\alpha g$–nbhd G of p such that $f(G) \subseteq M$. Hence, there exists a $\mathcal{N}_{ano}\, IS\alpha g$–open set H in $(U, T_R(X), I)$ such that $p \in H \subseteq G$ and so $f(H) \subseteq f(G) \subseteq M$.

$(7) \Leftrightarrow (4)$. Suppose that (4) holds and let G be a subset of $(U, T_R(X), I)$. Since $f(G) \subseteq \mathcal{N}_{ano}\text{-}Cl(f(G))$, we have $G \subseteq f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(G)))$. Since $\mathcal{N}_{ano}\text{-}Cl(f(G))$ is a $\mathcal{N}_{ano}$–closed set in $(V, \sigma_{R'}(Y), J)$, by assumption $f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(G)))$ is a $\mathcal{N}_{ano}\, IS\alpha g$–closed set containing G. Consequently, $\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(G) \subseteq f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(G)))$. Thus $f[\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(G)] \subseteq f[f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(G)))] \subseteq \mathcal{N}_{ano}\text{-}Cl(f(G))$.

Conversely, suppose that (7) holds for any subset G of $(U, T_R(X), I)$. Let H be a $\mathcal{N}_{ano}$–closed subset of $(V, \sigma_{R'}(Y), J)$. Then by assumption, $f[\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(H))] \subseteq \mathcal{N}_{ano}\text{-}Cl(f[f^{-1}(H)]) \subseteq \mathcal{N}_{ano}\text{-}Cl(H) = H$. i.e., $\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(H)) \subseteq f^{-1}(H)$ and so $f^{-1}(H)$ is $\mathcal{N}_{ano}\, IS\alpha g$–closed.

$(7) \Leftrightarrow (8)$. Suppose that (7) holds and B be any subset of $(V, \sigma_{R'}(Y), J)$. Then replacing A by $f^{-1}(B)$ in (7), we obtain $f(\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(B))) \subseteq \mathcal{N}_{ano}\text{-}Cl(f(f^{-1}(B))) \subseteq \mathcal{N}_{ano}\text{-}Cl(B)$, i.e., $\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N}_{ano}\text{-}Cl(B))$.

Conversely, suppose that (8) holds. Let $B = f(A)$) where A is a subset of $(U, T_R(X), I)$. Then we have, $\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(A) \subseteq \mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(f^{-1}(B)) \subseteq f^{-1}(\mathcal{N}_{ano}\text{-}Cl(f(A)))$ and so $f(\mathcal{N}_{ano}\, IS\alpha g\text{-}Cl(A)) \subseteq \mathcal{N}_{ano}\text{-}Cl(f(A))$. This completes the proof of the theorem.

4 Nano Ideal Semi $-\alpha-$Generalized $-$Irresolute Mappings

In this section $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}-$*function* is defined, and its characterizations and properties concerning $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt$ and $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}-Cl$ of sets are derived.

Definition 4.1. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is called $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-$*irresolute* if the inverse image $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ for each $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set H in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$.

Theorem 4.2. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute if and only if the inverse image of every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set in V is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in U.

Proof. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute. Let H be $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in V. Then H^{C} is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open in V. Since f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{Ig}^*\alpha-$irresolute $f^{-1}(\mathrm{H}^{C})=(f^{-1}(\mathrm{H}))^{C}= f^{-1}(\mathrm{V})-f^{-1}(\mathrm{H})=\mathrm{U}-f^{-1}(\mathrm{H})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open in U. Therefore $f^{-1}(\mathrm{H})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in U. Thus, the inverse image of every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set in V is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in U.

Conversely, let the inverse image of every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set in V is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in U. Let B be a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in V. Then B^{C} is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in V. By our assumption $f^{-1}(\mathrm{B}^{C})=(f^{-1}(\mathrm{B}))^{C}=f^{-1}(\mathrm{V})-f^{-1}(\mathrm{B})=\mathrm{U}-f^{-1}(\mathrm{B})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in U. Therefore $f^{-1}(\mathrm{B})$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open in U. Thus the inverse image of every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in V is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open in U. Hence f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute.

Example 4.3. Let $\mathrm{W}=\{a,b,c,d\}$, $\mathrm{W}/\mathrm{R}'=\{\{\{a\},\{b,c\},\{d\}\}\}$; $\mathrm{Z}=\{a,d\}$ and $\mathcal{N}_{\mathrm{ano}}$ $-$Ideal $\mathrm{J}=\{\phi,\{a\}\}$. $\mathcal{N}_{\mathrm{ano}}$ $-$Topology $\Omega_{\mathrm{R}'}(\mathrm{Z})=\{\phi,\mathrm{W},\{a,d\}\}$. Then the $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed sets are ϕ, $\{a\}$, $\{b,c\}$, $\{a,b,c\}$, $\{b,c,d\}$. Consider $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ as given in Example 3.9. Now define $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{W},\Omega_{\mathrm{R}'}(Z),\mathrm{J})$ by $f(1)=d, f(2)=b, f(3)=a, f(4)=c.$ We notice that

$$f^{-1}(\{a\})=\{3\}, f^{-1}(\{b,c\})=\{2,4\}, f^{-1}(\{a,b,c\})=\{2,3,4\}, f^{-1}(\{b,c,d\})=\{1,2,4\}, f(\phi)=\phi, f^{-1}(\mathrm{W})=\mathrm{U}$$

are $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed sets in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. as stated in Example 3.9. Then by Theorem 4.2, it follows that f is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute function.

Theorem 4.4. Every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute map is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$continuous but not conversely.

Proof. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute map. Let H be a $\mathcal{N}_{\mathrm{ano}}-$closed set of $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Then H is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed. Since f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute, then $f^{-1}(\mathrm{H})$ is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set of $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Therefore f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$continuous.

Example 4.5. Let $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ be a $\mathcal{N}_{\mathrm{ano}}$ $-$ideal $-$Top. $-$Spaces as given in Example 3.9 and $(\mathrm{W},\Omega_{\mathrm{R}^*}(\mathrm{Z}),\mathrm{J})$ as given in Example 4.13. Define $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\to(\mathrm{W},\Omega_{\mathrm{R}^*}(\mathrm{Z}),\mathrm{J})$ by $f(1)=a, f(2)=b, f(3)=d, f(4)=c.$ It is clear that $\{\alpha\}$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set of $(\mathrm{W},\Omega_{\mathrm{R}^*}(\mathrm{Z}),\mathrm{J})$ but $f^{-1}(\{a\})=\{1\}$ is not $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set of $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Thus f is not

$\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute map. We notice that $f^{-1}(\{a,d\})=\{1,3\}$ is $\mathcal{N}_{\text{ano}}$-open in $(U, T_R(X), I)$ and therefore f is $\mathcal{N}_{\text{ano}}$-continuous and hence f is also a $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-continuous map.

Theorem 4.6. Let $f:(U, T_R(X), I)\to(V, \sigma_{R'}(Y), J)$ be a function. Then the following statements are equivalent.

(i) f is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute.

(ii) For every subset A of U, $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left[f^{-1}\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right)\right]=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$.

(iii) For every subset B of V, $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left[f^{-1}\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(B)\right)\right]=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$.

Proof. (i)$\Leftrightarrow$(ii). Let f be $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute and $A\subseteq U$. Then $f(A)\subseteq V$. $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))$ is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-closed in V. Since f is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute, $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$ is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-closed in U. Therefore we obtain $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$.

Conversely, suppose that we have $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]\right)=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$ for every subset A of U, Let H be a $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-closed set in V. Since $f^{-1}(H)\subseteq U$,
$\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{--}Cl\left(f\left(f^{-1}(H)\right)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f\left(f^{-1}(H)\right)\right)\right]$. That is $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}(H)\right)=f^{-1}(H)$ implies that $f^{-1}(H)$ is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-closed in U. Hence f is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute.

(ii)$\Leftrightarrow$(iii). Assume (ii) holds. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ in (ii) we have $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f\left(f^{-1}(B)\right)\right)\right]\right)=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f\left(f^{-1}(B)\right)\right)\right]$. That is $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(B)\right]\right)=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(B)\right]$.

Conversely, suppose (iii) holds. Let A be any subset of U. Then $f(A)\subseteq V$. Let $B=f(A)$. Then we have $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl\left(f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Cl(f(A))\right]$ for every subset A of U.

Theorem 4.7. Let $f:(U, T_R(X), I)\to(V, \sigma_{R'}(Y), J)$ be a function. Then the following statements are equivalent.

(i) f is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute.

(ii) For every subset A of U, $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Int\left[f^{-1}\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Int(f(A))\right)\right]=$ $f^{-1}\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Int(f(A))\right]$.

(iii) For every subset B of V, $\mathcal{N}_{ano}\,\mathrm{I}g^{*}\alpha\text{-}Int\left[f^{-1}\left(\mathcal{N}_{ano}\,\mathrm{I}g^{*}\alpha\text{-}Int(B)\right)\right]=$ $f^{-1}\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Int(B)\right)$.

Proof. (i)$\Leftrightarrow$(ii). Let f be $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-irresolute and $A\subseteq U$. Then $f(A)\subseteq V$. $\mathcal{N}_{ano}\,\mathrm{IS}\alpha g\text{-}Int(f(A))$ is $\mathcal{N}_{\text{ano}}\,\mathrm{IS}\alpha g$-open in V. Since f is

$\mathcal{N}_{ano}\,IS\alpha g$-irresolute, $f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(A\right)\right)\right]$ is $\mathcal{N}_{ano}\,IS\alpha g$-open in U. Therefore, we obtain $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(A\right)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(A\right)\right)\right]$.

Conversely, suppose that we have $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(A\right)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(A\right)\right)\right]$ for every subset A of U, Let G be a $\mathcal{N}_{ano}\,IS\alpha g$-open set in V. Since $f^{-1}(G)\subseteq U$, $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(f^{-1}(G)\right)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(f^{-1}(G)\right)\right)\right]$. That is $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}(G)\right)=f^{-1}(G)$ implies that $f^{-1}(G)$ is $\mathcal{N}_{ano}\,IS\alpha g$-open in U. Hence f is $\mathcal{N}_{ano}\,IS\alpha g$-irresolute

$(ii)\Leftrightarrow(iii)$. Assume (ii) holds. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ in (ii) we get $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(f^{-1}(B)\right)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f\left(f^{-1}(B)\right)\right)\right]$. That is $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int(B)\right]\right)=$ 2 pcf $f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int(B)\right]$.

Conversely, suppose (iii) holds. Let A be any subset of U. Then $f(A)\subseteq V$. Let $B=f(A)$. Then $\mathcal{N}_{ano}\,IS\alpha g-Int\left(f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f(A)\right)\right]\right)=f^{-1}\left[\mathcal{N}_{ano}\,IS\alpha g-Int\left(f(A)\right)\right]$ for every subset A of U.

Theorem 4.8. Let $f:\left(U,T_R(X),I\right)\rightarrow\left(V,\sigma_{R'}(Y),J\right)$ be a $\mathcal{N}_{ano}\,IS\alpha g$-irresolute function. Then f is a $\mathcal{N}_{ano}\,IS\alpha g$-continuous function.

Proof. Let $f:\left(U,T_R(X),I\right)\rightarrow\left(V,\sigma_{R'}(Y),J\right)$ be $\mathcal{N}_{ano}\,IS\alpha g$-irresolute. Let G be any $\mathcal{N}_{ano}$-open set in V, Then G is $\mathcal{N}_{ano}\,IS\alpha g$-open in V, Since f is $\mathcal{N}_{ano}\,IS\alpha g$-irresolute, $f^{-1}(G)$ is $\mathcal{N}_{ano}\,IS\alpha g$-open in U, Thus, the inverse image of every $\mathcal{N}_{ano}$-open set in V is $\mathcal{N}_{ano}\,IS\alpha g$-open in U, Therefore any $\mathcal{N}_{ano}\,IS\alpha g$-irresolute function is a $\mathcal{N}_{ano}\,IS\alpha g$-continuous function.

Theorem 4.9. Consider two functions $f:\left(U,T_R(X),I\right)\rightarrow\left(V,\sigma_{R'}(Y),J\right)$ and $g:\left(V,\sigma_{R'}(Y),J\right)\rightarrow\left(W,\Omega_{R''}(Z),K\right)$. Then the following statements are true.

(1). Let f be a $\mathcal{N}_{ano}\,IS\alpha g$-irresolute function and g be a $\mathcal{N}_{ano}\,IS\alpha g$-continuous function. Then show that $gof:\left(U,T_R(X),I\right)\rightarrow\left(W,\Omega_{R''}(Z),K\right)$ is a $\mathcal{N}_{ano}\,IS\alpha g$-continuous function.

(2). gof is $\mathcal{N}_{ano}\,IS\alpha g$-irresolute if both f and g are $\mathcal{N}_{ano}\,IS\alpha g$-irresolute.

Proof. (1). Let G be $\mathcal{N}_{ano}$-open in W. Since g is $\mathcal{N}_{ano}\,IS\alpha g$-continuous $g^{-1}(G)$ is $\mathcal{N}_{ano}\,IS\alpha g$-open in V. Since f is $\mathcal{N}_{ano}\,IS\alpha g$-irresolute, $f^{-1}\left(g^{-1}(G)\right)=(gof)^{-1}(G)$ is $\mathcal{N}_{ano}\,IS\alpha g$-open in U. Thus the inverse image of every $\mathcal{N}_{ano}$-open set in W is $\mathcal{N}_{ano}\,IS\alpha g$-open in U. Therefore gof is $\mathcal{N}_{ano}\,IS\alpha g$-continuous.

(2). Easy.

Theorem 4.10. If $f:\left(U,T_R(X),I\right)\rightarrow\left(V,\sigma_{R'}(Y),J\right)$ is a $\mathcal{N}_{ano}\,IS\alpha g$-irresolute function and $g:\left(V,\sigma_{R'}(Y),J\right)\rightarrow\left(W,\Omega_{R''}(Z),K\right)$ is a $\mathcal{N}_{ano}$-continuous function, then $gof:\left(U,T_R(X),I\right)\rightarrow\left(W,\Omega_{R''}(Z),K\right)$ is $\mathcal{N}_{ano}\,IS\alpha g$-continuous.

Proof. Proof is like Theorem (4.9) since $\mathcal{N}_{ano}$-open set is a $\mathcal{N}_{ano}\,IS\alpha g$-open set.

Theorem 4.11. If $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is a $\mathcal{N}_{ano}IS\alpha g$–irresolute function and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),K)$ is a $\mathcal{N}_{ano}$–contra–continuous function, then $gof:(U,T_R(X),I)\to(W,T_{R''}(Z),K)$ is $\mathcal{N}_{ano}IS\alpha g$–continuous.

Proof. Proof is like Theorem (4.9) since $\mathcal{N}_{ano}$–closed set is a $\mathcal{N}_{ano}IS\alpha g$–closed set.

Theorem 4.12. If $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),K)$ are $\mathcal{N}_{ano}IS\alpha g$–irresolutes, then $gof:(U,T_R(X),I)\to(W,\Omega_{R''}(Z),K)$ is $\mathcal{N}_{ano}IS\alpha g$–irresolute.

Proof. Let G be $\mathcal{N}_{ano}$–open in W. Since g is $\mathcal{N}_{ano}IS\alpha g$–irresolute $g^{-1}(G)$ is $\mathcal{N}_{ano}IS\alpha g$–open in V. Since f is $\mathcal{N}_{ano}IS\alpha g$–irresolute, $f^{-1}(g^{-1}(G))=(gof)^{-1}(G)$ is $\mathcal{N}_{ano}IS\alpha g$–open in U. Thus, the inverse image of every $\mathcal{N}_{ano}IS\alpha g$–open set in W is $\mathcal{N}_{ano}IS\alpha g$–open in U. Therefore gof is $\mathcal{N}_{ano}IS\alpha g$–irresolute.

5 Nano Ideal Semi –α –Generalized –Open and Closed Mappings

In this section $\mathcal{N}_{ano}IS\alpha g$–open mappings and $\mathcal{N}_{ano}IS\alpha g$–closed Mappings are defined, and some of their characterizations are presented.

Definition 5.1. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is said to be a $\mathcal{N}_{ano}IS\alpha g$–open–map on U if the image of every $\mathcal{N}_{ano}$–open set in $(U,T_R(X),I)$ is a $\mathcal{N}_{ano}IS\alpha g$–open set in $(V,T_{R'}(Y),J)$.

Definition 5.2. A mapping $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is said to be a $\mathcal{N}_{ano}IS\alpha g$–closed–map on U if the image of every $\mathcal{N}_{ano}$–closed set in $(U,T_R(X))$ is a $\mathcal{N}_{ano}IS\alpha g$–closed set in $(V,\sigma_{R'}(Y),J)$.

Definition 5.3. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is said to be a Strongly $\mathcal{N}_{ano}IS\alpha g$–closed map on U if the image of every $\mathcal{N}_{ano}IS\alpha g$–closed set in $(U,T_R(X),I)$ is a $\mathcal{N}_{ano}IS\alpha g$–closed set in $(V,\sigma_{R'}(Y),J)$.

Definition 5.4. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is said to be a Strongly $\mathcal{N}_{ano}IS\alpha g$–open map on U if the image of every $\mathcal{N}_{ano}IS\alpha g$–open set in $(U,T_R(X),I)$ is a $\mathcal{N}_{ano}IS\alpha g$–open set in $(V,\sigma_{R'}(Y),J)$.

Theorem 5.5. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g$–closed if and only if for each subset A of V and for each $\mathcal{N}_{ano}$–open set G of $(U,T_R(X),I)$ containing $f^{-1}(A)$, there is a $\mathcal{N}_{ano}IS\alpha g$–open set B of $(V,\sigma_{R'}(Y),J)$ such that $A\subseteq B$ and $f^{-1}(B)\subseteq G$.

Proof. Let A be a subset of $(V,\sigma_{R'}(Y),J)$ and G be a $\mathcal{N}_{ano}$–open set of $(U,T_R(X),I)$ such that $f^{-1}(A)\subseteq G$. Then $U-G$ is a $\mathcal{N}_{ano}$–closed set of U. Since f is $\mathcal{N}_{ano}IS\alpha g$–closed, $f(U-G)$ is $\mathcal{N}_{ano}IS\alpha g$–closed in $(V,\sigma_{R'}(Y),J)$. Now $B=V-f(U-G)$ is a $\mathcal{N}_{ano}IS\alpha g$–open set containing A in V such that $f^{-1}(B)\subseteq G$.

Conversely, let H be a $\mathcal{N}_{ano}$–closed set of $(U,T_R(X),I)$, then $f^{-1}(V-f(H))\subseteq U-H$ and $U-H$ is $\mathcal{N}_{ano}$–open. By our assumption, there is a $\mathcal{N}_{ano}IS\alpha g$–open set B of $(V,\sigma_{R'}(Y),J)$ such that $V-f(H)\subseteq B$ and $f^{-1}(B)\subseteq U-H$. Hence $V-B\subseteq f(H)$ and $H\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(H)\subseteq f(U-f^{-1}(B))\subseteq V-B$ which implies $f(H)=V-B$. Since $V-B$ is $\mathcal{N}_{ano}IS\alpha g$−closed, $f(H)$ is a $\mathcal{N}_{ano}IS\alpha g$−closed set in $(V,\sigma_{R'}(Y),J)$. That is $f(H)$ is $\mathcal{N}_{ano}IS\alpha g$−closed in V for every $\mathcal{N}_{ano}$−closed set H of U. Hence f is a $\mathcal{N}_{ano}IS\alpha g$−closed −map.

Theorem 5.6. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g$−open if and only if for each subset A of V and for each $\mathcal{N}_{ano}$−closed set H of $(U,T_R(X),I)$ containing $f^{-1}(A)$, there is a $\mathcal{N}_{ano}IS\alpha g$−closed set B of $(V,\sigma_{R'}(Y),J)$ such that $A\subseteq B$ and $f^{-1}(B)\subseteq H$.

Proof. Let A be a subset of $(V,\sigma_{R'}(Y),J)$ and H be a $\mathcal{N}_{ano}$−closed set of $(U,T_R(X),I)$ such that $f^{-1}(A)\subseteq H$. Then $U-H$ is a $\mathcal{N}_{ano}$−open set of U. Since f is $\mathcal{N}_{ano}IS\alpha g$−open, $f(U-H)$ is $\mathcal{N}_{ano}IS\alpha g$−open in $(V,\sigma_{R'}(Y),J)$. Now $B=V-f(U-H)$ is a $\mathcal{N}_{ano}IS\alpha g$−closed set containing A in V such that $f^{-1}(B)\subseteq H$. Conversely let G be a $\mathcal{N}_{ano}$−open set of $(U,T_R(X),I)$, then $f^{-1}(V-f(G))\subseteq U-G$ and $U-G$ is $\mathcal{N}_{ano}$−closed. By our assumption there is a $\mathcal{N}_{ano}IS\alpha g$−closed set B of $(V,\sigma_{R'}(Y),J)$ such that $V-f(G)\subseteq B$ and $f^{-1}(B)\subseteq U-G$. Hence $V-B\subseteq f(G)$ and $G\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(G)\subseteq f(U-f^{-1}(B))\subseteq V-B$ which implies $f(G)=V-B$. Since $V-B$ is $\mathcal{N}_{ano}IS\alpha g$−open, $f(G)$ is a $\mathcal{N}_{ano}IS\alpha g$−open set in $(V,\sigma_{R'}(Y),J)$. That is $f(G)$ is $\mathcal{N}_{ano}IS\alpha g$−open in V for every $\mathcal{N}_{ano}$−open set G of U. Hence f is a $\mathcal{N}_{ano}IS\alpha g$−open −map.

Theorem5.7. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ be a function. Then the following statements are equivalent

(i) f is strongly $\mathcal{N}_{ano}IS\alpha g$−closed.

(ii) For every subset A of V and every $\mathcal{N}_{ano}IS\alpha g$−open G of $(U,T_R(X),I)$ containing $f^{-1}(A)$, there is a $\mathcal{N}_{ano}IS\alpha g$−open set B of $(V,\sigma_{R'}(Y),J)$ with $A\subseteq B$ and $f^{-1}(B)\subseteq G$.

Proof. Let A be a subset of $(V,\sigma_{R'}(Y),J)$ and G be a $\mathcal{N}_{ano}IS\alpha g$−open set of $(U,T_R(X),I)$ such that $f^{-1}(A)\subseteq G$. Then $U-G$ is a $\mathcal{N}_{ano}IS\alpha g$−closed set of U. Since f is strongly $\mathcal{N}_{ano}IS\alpha g$−closed, $f(U-G)$ is $\mathcal{N}_{ano}IS\alpha g$−closed in $(V,\sigma_{R'}(Y),J)$. Now $B=V-f(U-G)$ is a $\mathcal{N}_{ano}IS\alpha g$−open set containing A in V such that $f^{-1}(B)\subseteq G$.

Conversely, let H be a $\mathcal{N}_{ano}IS\alpha g$−closed set of $(U,T_R(X),I)$, then $f^{-1}(V-f(H))\subseteq U-H$ and $U-H$ is $\mathcal{N}_{ano}IS\alpha g$−open. By our assumption, there is a $\mathcal{N}_{ano}IS\alpha g$−open set B of $(V,\sigma_{R'}(Y),J)$ such that $V-f(H)\subseteq B$ and $f^{-1}(B)\subseteq U-H$. Hence $V-B\subseteq f(H)$ and $H\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(H)\subseteq f(U-f^{-1}(B))\subseteq V-B$ which implies $f(H)=V-B$. Since $V-B$ is $\mathcal{N}_{ano}IS\alpha g$−closed, $f(H)$ is a $\mathcal{N}_{ano}IS\alpha g$−closed set in $(V,\sigma_{R'}(Y),J)$. That is $f(H)$ is $\mathcal{N}_{ano}IS\alpha g$−closed in V for every $\mathcal{N}_{ano}IS\alpha g$−closed set H of U. Hence f is a strongly $\mathcal{N}_{ano}IS\alpha g$−closed −map.

Theorem 5.8. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ be a function. Then the following statements are equivalent

(i) f is strongly $\mathcal{N}_{ano}IS\alpha g$−open.

(ii) For every subset A of V and every $\mathcal{N}_{ano}IS\alpha g$-closed set H of $(U,T_R(X),I)$ containing $f^{-1}(A)$, there is a $\mathcal{N}_{ano}Ig^*\alpha$-closed set B of $(V,\sigma_{R'}(Y),J)$ with $A\subseteq B$ and $f^{-1}(B)\subseteq H$.

Proof. Let A be a subset of $(V,\sigma_{R'}(Y),J)$ and H be a $\mathcal{N}_{ano}IS\alpha g$-closed set of $(U,T_R(X))$ such that $f^{-1}(A)\subseteq H$. Then $U-H$ is a $\mathcal{N}_{ano}IS\alpha g$-open set of U. Since f is strongly $\mathcal{N}_{ano}IS\alpha g$-open, $f(U-H)$ is $\mathcal{N}_{ano}IS\alpha g$-open in $(V,\sigma_{R'}(Y),J)$. Now $B=V-f(U-H)$ is a $\mathcal{N}_{ano}IS\alpha g$-closed set containing A in V such that $f^{-1}(B)\subseteq H$.

Conversely, let G be a $\mathcal{N}_{ano}IS\alpha g$-open set of $(U,T_R(X),I)$, then $f^{-1}(V-f(G))\subseteq U-G$ and $U-G$ is $\mathcal{N}_{ano}IS\alpha g$-closed. By our assumption there is a $\mathcal{N}_{ano}IS\alpha g$-closed set B of $(V,\sigma_{R'}(Y),J)$ such that $V-f(G)\subseteq B$ and $f^{-1}(B)\subseteq U-G$. Hence $V-B\subseteq f(G)$ and $G\subseteq U-f^{-1}(B)$. Thus $V-B\subseteq f(G)\subseteq f(U-f^{-1}(B))\subseteq V-B$ which implies $f(G)=V-B$. Since $V-B$ is $\mathcal{N}_{ano}IS\alpha g$-open, $f(G)$ is a $\mathcal{N}_{ano}IS\alpha g$-open set in $(V,\sigma_{R'}(Y),J)$. That is $f(G)$ is $\mathcal{N}_{ano}IS\alpha g$-open in V for every $\mathcal{N}_{ano}IS\alpha g$-open set G of U. Hence f is a strongly $\mathcal{N}_{ano}IS\alpha g$-open-map.

Theorem 5.9. A map $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g$-closed if and only if $\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))\subseteq f(\mathcal{N}_{ano}Cl(A))$ for every subset A of U.

Proof. Suppose that f is $\mathcal{N}_{ano}IS\alpha g$-closed and $A\subseteq U$. Then $\mathcal{N}_{ano}\text{-}Cl(A)$ is $\mathcal{N}_{ano}$-closed in U and so $f(\mathcal{N}_{ano}\text{-}Cl(A))$ is $\mathcal{N}_{ano}IS\alpha g$-closed in $(V,\sigma_{R'}(Y),J)$. We have $f(A)\subseteq f(\mathcal{N}_{ano}\text{-}Cl(A))$ and $\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl[f(\mathcal{N}_{ano}\text{-}Cl(A))]=f(\mathcal{N}_{ano}\text{-}Cl(A))$.

Conversely, let A be any $\mathcal{N}_{ano}$-closed set in U. Then $A=\mathcal{N}_{ano}\text{-}Cl(A)$ and so $f(A)=f(\mathcal{N}_{ano}\text{-}Cl(A))\supseteq\mathcal{N}_{ano}Ig^*\alpha\text{-}Cl(f(A))$, by hypothesis. We have $f(A)\subseteq\mathcal{N}_{ano}Ig^*\alpha\text{-}Cl(f(A))$. Therefore $f(A)=\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))$. i.e., $f(A)$ is $\mathcal{N}_{ano}IS\alpha g$-closed and hence f is $\mathcal{N}_{ano}IS\alpha g$-closed.

Theorem 5.10. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ be a function such that $\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))\subseteq f(\mathcal{N}_{ano}\text{-}Cl(A))$ for every subset A of U. Then the image $f(A)$ of a $\mathcal{N}_{ano}$-closed set A in U is $\mathcal{N}_{ano}IS\alpha g$-closed in V.

Proof. Let A be a $\mathcal{N}_{ano}$-closed set in U. Then by hypothesis, $\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))\subseteq f(\mathcal{N}_{ano}\text{-}Cl(A))=f(A)$ and so $\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f(A))=f(A)$. Therefore $f(A)$ is $\mathcal{N}_{ano}IS\alpha g$-closed in V.

Theorem 5.11. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ be a $\mathcal{N}_{ano}$-closed function and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),K)$ be a $\mathcal{N}_{ano}Ig^*\alpha$-closed function, then their composition function $gof:(U,T_R(X),I)\to(W,\Omega_{R''}(Z),K)$ is $\mathcal{N}_{ano}IS\alpha g$-closed function.

Proof. Let A be a $\mathcal{N}_{ano}$-closed set of U. Then by assumption $f(A)$ is $\mathcal{N}_{ano}$-closed in V. Since g is $\mathcal{N}_{ano}IS\alpha g$-closed function, $g(f(A))$ is $\mathcal{N}_{ano}IS\alpha g$-closed set in W. i.e., $(gof)(A)$ is $\mathcal{N}_{ano}IS\alpha g$-closed set in W and so gof is a $\mathcal{N}_{ano}IS\alpha g$-closed function.

Theorem 5.12. Let $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ and $g:(V,T_{R'}(Y),J)\to(W,\Omega_{R''}(Z),K)$ be two functions such that

$g o f : (U, T_R(X), I) \to (W, \Omega_{R''}(Z), K)$ is a $\mathcal{N}_{ano}$ ISαg−closed function. Then the next conditions are true.

(i) If f is $\mathcal{N}_{ano}$ −continuous and surjective, then g is $\mathcal{N}_{ano}$ ISαg−closed.

(ii) If g is $\mathcal{N}_{ano}$ ISαg−irresolute and injective, then f is $\mathcal{N}_{ano}$ ISαg−closed.

(iii) If g is strongly $\mathcal{N}_{ano}$ ISαg−continuous and injective, then f is $\mathcal{N}_{ano}$ −closed.

Proof. (i) Let B be a $\mathcal{N}_{ano}$ −closed set of V. Since f is $\mathcal{N}_{ano}$ −continuous, $f^{-1}(B)$ is $\mathcal{N}_{ano}$ −closed in U and since $g o f$ is $\mathcal{N}_{ano}$ ISαg−closed, $(g o f)(f^{-1}(B))$ is $\mathcal{N}_{ano}$ ISαg−closed in W. That is $g(B)$ is $\mathcal{N}_{ano}$ ISαg−closed in W, since f is surjective. Therefore g is a $\mathcal{N}_{ano}$ ISαg−closed function.

(ii) Let A be a $\mathcal{N}_{ano}$ −closed set of U. Since $g o f$ is $\mathcal{N}_{ano}$ ISαg−closed, $(g o f)(A)$ is $\mathcal{N}_{ano}$ ISαg−closed in W. Since g is $\mathcal{N}_{ano}$ ISαg−irresolute, $g^{-1}((g o f)(A))$ is $\mathcal{N}_{ano}$ ISαg−closed set in V. That is $f(A)$ is $\mathcal{N}_{ano}$ ISαg−closed in V, since g is injective. Thus f is a $\mathcal{N}_{ano}$ ISαg−closed function.

(iii) Let A be a $\mathcal{N}_{ano}$ −closed set of U. Since $g o f$ is $\mathcal{N}_{ano}$ ISαg−closed, $(g o f)(A)$ is $\mathcal{N}_{ano}$ ISαg−closed in W. Since g is strongly $\mathcal{N}_{ano}$ ISαg−continuous, $g^{-1}((g o f)(A))$ is $\mathcal{N}_{ano}$ −closed in V. That is $f(A))$ is $\mathcal{N}_{ano}$ −closed set in V, since g is injective. Therefore f is a $\mathcal{N}_{ano}$ −closed function.

Theorem 5.13. Let $f : (U, T_R(X), I) \to (V, \sigma_{R'}(Y), J)$ be a function. Then the next conditions are equivalent:

(i) f is a $\mathcal{N}_{ano}$ ISαg−open function.

(ii) For a subset A of U, $f(\mathcal{N}_{ano}\text{-}Int(A)) \subseteq \mathcal{N}_{ano}\,IS\alpha g\text{-}Int(f(A))$.

(iii) For each $x \in U$ and for each $\mathcal{N}_{ano}$ −nbhd G of x in U, there exists a $\mathcal{N}_{ano}$ ISαg−nbhd M of $f(x)$ in V such that $M \subseteq f(G)$.

Proof. $(i) \Rightarrow (ii)$. Suppose f is $\mathcal{N}_{ano}$ ISαg−open. Let $A \subseteq U$. Then $\mathcal{N}_{ano}\text{-}Int(A)$ is $\mathcal{N}_{ano}$ −open in U and so $f(\mathcal{N}_{ano}\text{-}Int(A))$ is $\mathcal{N}_{ano}$ ISαg−open in V. We have $f(\mathcal{N}_{ano}\text{-}Int(A)) \subseteq f(A)$. Therefore $f(\mathcal{N}_{ano}\text{-}Int(A)) \subseteq \mathcal{N}_{ano}\,Ig^*\alpha\text{-}Int(f(A))$.

$(ii) \Rightarrow (iii)$. Suppose (ii) holds. Let $x \in U$ and G be an arbitrary $\mathcal{N}_{ano}$ −nbhd of x in U. Then there exists a $\mathcal{N}_{ano}$ −open set H such that $x \in H \subseteq G$. By assumption, we have $f(H) = f(\mathcal{N}_{ano}\text{-}Int(H)) \subseteq \mathcal{N}_{ano}\,IS\alpha g\text{-}Int(f(H))$. Thus, $f(H) = \mathcal{N}_{ano}\,IS\alpha g\text{-}Int(f(H))$. We have $f(H)$ is $\mathcal{N}_{ano}$ ISαg−open in V. Further, $f(x) \in f(H) \subseteq f(G)$ and so (iii) holds, by taking $M = f(H)$.

$(iii) \Rightarrow (i)$. Suppose (iii) holds. Let G be any $\mathcal{N}_{ano}$ −open set in U, $x \in G$ and $f(x) = y$. Then $y \in f(G)$ and for each $y \in f(G)$, by assumption, there exists a $\mathcal{N}_{ano}$ ISαg−nbhd M_y of y in V such that $M_y \subseteq f(G)$. Since M_y is a $\mathcal{N}_{ano}$ ISαg−nbhd of y, there exists a $\mathcal{N}_{ano}$ ISαg−open set P_y in V such that $y \in P_y \subseteq M_y$. Therefore, $f(G) = \bigcup\{P_y : y \in f(G)\}$ is a $\mathcal{N}_{ano}$ ISαg−open set in V. Thus f is a $\mathcal{N}_{ano}$ ISαg−open function.

Theorem 5.14. Let $f : (U, T_R(X), I) \to (V, \sigma_{R'}(Y), J)$ be any bijection. Then the following statements are equivalent.

(i) f^{-1} is $\mathcal{N}_{ano}$ ISαg−continuous.

(ii) f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open.

(iii) f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed.

Proof. $(i)\Rightarrow(ii)$. Let A be $\mathcal{N}_{ano}$–open set in U. Then $\mathrm{U}-\mathrm{A}$ is $\mathcal{N}_{ano}$–closed in U. Since f^{-1} is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous, $\left(f^{-1}\right)^{-1}(\mathrm{U}-\mathrm{A})=f(\mathrm{U}-\mathrm{A})=\mathrm{V}-f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in V. Then $f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open in V. Hence f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open.

$(ii)\Rightarrow(iii)$. Let A be $\mathcal{N}_{ano}$–closed set in U. Then $\mathrm{U}-\mathrm{A}$ is $\mathcal{N}_{ano}$–open in U. Since f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open, $f(\mathrm{U}-\mathrm{A})=\mathrm{V}-f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open in V. Then $f(\mathrm{A})$ $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in V. Hence f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed.

$(iii)\Rightarrow(i)$. Let A be $\mathcal{N}_{ano}$–closed in U. Since $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed, $f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in V. *i.e.,* $\left(f^{-1}\right)^{-1}(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in V. Therefore f^{-1} is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous.

6 Nano Ideal Semi –α –Generalized –Homeomorphisms

In this section, we define and study the concept of nano ideal semi-α generalized-homeomorphisms (briefly, $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–homeomorphisms) in $\mathcal{N}_{ano}$–Ideal–Top–Space $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ and obtain some of its properties and characterizations.

Definition 6.1. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is said to be $\mathcal{N}_{ano}$–homeomorphisms if f is one-to-one & onto, $\mathcal{N}_{ano}$–open and $\mathcal{N}_{ano}$–continuous.

Definition 6.2. A map $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is said to be $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–homeomorphisms if f is one-to-one & onto, $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open and $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous.

Theorem 6.3. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be a one-to-one onto mapping. Then f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–homeomorphism if and only if f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed and $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous.

Proof. Let $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ be a $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–homeomorphism. Then f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous. Let A be an arbitrary $\mathcal{N}_{ano}$–closed set in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Then $\mathrm{U}-\mathrm{A}$ is $\mathcal{N}_{ano}$–open. Since f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open, $f(\mathrm{U}-\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. That is, $\mathrm{V}-f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ for every $\mathcal{N}_{ano}$–closed set A in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$ implies that $f(\mathrm{A})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in $((\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Hence $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed.

Conversely, let f be $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed and $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–continuous function. Let G be a $\mathcal{N}_{ano}$–open set in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Then $\mathrm{U}-\mathrm{G}$ is $\mathcal{N}_{ano}$–closed in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Since f is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed, $f(\mathrm{U}-\mathrm{G})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. That is, $f(\mathrm{U}-\mathrm{G})=\mathrm{V}-f(\mathrm{G})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–closed in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$. Hence $f(\mathrm{G})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–open in $(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ for every $\mathcal{N}_{ano}$–open set G in $(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})$. Thus, f is $\mathcal{N}_{ano}$ $\mathrm{Ig}^*\alpha$–open and so $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{ano}$ $\mathrm{IS}\alpha\mathrm{g}$–homeomorphism.

Theorem 6.4.: Let $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ be a bijective and $\mathcal{N}_{ano}IS\alpha g-$continuous map. Then the following statements are equivalent.

(i) f is a $\mathcal{N}_{ano}IS\alpha g-$open map.

(ii) f is a $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

(iii) f is a $\mathcal{N}_{ano}IS\alpha g-$closed map.

Proof. $((i)\Rightarrow(ii)$. By the given hypothesis, the map $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is bijective $\mathcal{N}_{ano}IS\alpha g-$continuous and $\mathcal{N}_{ano}IS\alpha g-$open. Hence $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

$(ii)\Rightarrow(iii)$. Let A be a $\mathcal{N}_{ano}-$closed set in $(U,T_R(X),I)$. Then A^C is $\mathcal{N}_{ano}-$open in $(U,T_R(X),I)$. By assumption, $f(A^C)$ is $\mathcal{N}_{ano}IS\alpha g-$open in $(V,\sigma_{R'}(Y),J)$. That is $f(A^C)=(f(A))^C=V-f(A)$ is $\mathcal{N}_{ano}IS\alpha g-$open in $(V,\sigma_{R'}(Y),J)$ and hence $f(A)$ is $\mathcal{N}_{ano}IS\alpha g-$closed in $(V,\sigma_{R'}(Y),J)$ for every $\mathcal{N}_{ano}-$closed set A in $(U,T_R(X),I)$. Hence the function $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g-$closed map.

$(iii)\Rightarrow(i)$. Let B be a $\mathcal{N}_{ano}-$open set in $(U,T_R(X),I)$. Then B^C is $\mathcal{N}_{ano}-$closed set in $(U,T_R(X),I)$. By the given hypothesis, $f(B^C)$ is $\mathcal{N}_{ano}IS\alpha g-$closed in $(V,\sigma_{R'}(Y),J)$. But $f(B^C)=(f(B))^C=V-f(B)$ is $\mathcal{N}_{ano}IS\alpha g-$closed. Therefore $f(B)$ is $\mathcal{N}_{ano}IS\alpha g-$open in $(V,\sigma_{R'}(Y),J)$ for every $\mathcal{N}_{ano}-$open set B in $(U,T_R(X),I)$. Hence $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g-$open map.

Theorem 6.5. Every $\mathcal{N}_{ano}-$homeomorphism is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

Proof. If $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}-$homeomorphism, by definition 6.1, f is bijective, $\mathcal{N}_{ano}-$continuous and $\mathcal{N}_{ano}-$open. Then $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g-$continuous and $\mathcal{N}_{ano}IS\alpha g-$open. Hence the function $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism. Every $\mathcal{N}_{ano}-$continuous function is $\mathcal{N}_{ano}IS\alpha g-$continuous and every $\mathcal{N}_{ano}-$open map is $\mathcal{N}_{ano}IS\alpha g-$open. Then f is bijective, $\mathcal{N}_{ano}IS\alpha g-$continuous and $\mathcal{N}_{ano}IS\alpha g-$open. Therefore f is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

Theorem 6.6. Let $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ be a $\mathcal{N}_{ano}\alpha I-$homeomorphism. Then prove that f is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

Proof. Let $f:(U,T_R(X),I)\rightarrow(V,\sigma_{R'}(Y),J)$ be a $\mathcal{N}_{ano}\alpha I-$homeomorphism. Then f is bijective, $\mathcal{N}_{ano}\alpha I-$continuous and $\mathcal{N}_{ano}\alpha I-$open. Let H be a $\mathcal{N}_{ano}-$open set in $(V,\sigma_{R'}(Y),J)$. Since f is $\mathcal{N}_{ano}\alpha I-$continuous, $f^{-1}(H)$ is $\mathcal{N}_{ano}\alpha I-$open set in $(U,T_R(X),I)$. Since every $\mathcal{N}_{ano}\alpha I-$open set is $\mathcal{N}_{ano}IS\alpha g-$open set, $f^{-1}(H)$ is $\mathcal{N}_{ano}IS\alpha g-$open in $(U,T_R(X),I)$ which implies that f is $\mathcal{N}_{ano}IS\alpha g-$continuous. Let G be a $\mathcal{N}_{ano}-$open set in $(U,T_R(X),I)$. Since f is $\mathcal{N}_{ano}\alpha I-$open, $f(G)$ is $\mathcal{N}_{ano}\alpha I-$open set in $(V,\sigma_{R'}(Y),J)$. Since every $\mathcal{N}_{ano}\alpha I-$open set is $\mathcal{N}_{ano}IS\alpha g-$open set, $f(G)$ is $\mathcal{N}_{ano}IS\alpha g-$open set in $(V,\sigma_{R'}(Y),J)$ which implies f is $\mathcal{N}_{ano}IS\alpha g-$open map. Therefore, consequently f is $\mathcal{N}_{ano}IS\alpha g-$homeomorphism.

Theorem 6.7. Let $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ be a bijective and $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$irresolute map. Then the following statements are equivalent.

$(\boldsymbol{i})$ f is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open map.

$(\boldsymbol{ii})$ f is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$homeomorphism.

$(\boldsymbol{iii})$ f is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed map.

Proof. Identical to the proof of Theorem 6.5.

Theorem 6.8. Let $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ be a bijective function. Then prove that f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$homeomorphism if and only if $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]=\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)$ for every subset A of $\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)$.

Proof. If $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$homeomorphisms, then f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$continuous and $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed. If $\mathrm{A}\subseteq\mathrm{U}$, Then $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]\subseteq\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)$ since f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$continuous. Since $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})$ is $\mathcal{N}_{\mathrm{ano}}-$closed in $\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)$ and f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed map, $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in $\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$. Also $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl\left[f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right)\right]=f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right)$. Since $\mathrm{A}\subseteq\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})$, $f(\mathrm{A})\subseteq f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]$ and hence $\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)\subseteq\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left[f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right)\right]=$ $f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right)$ Thus $\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)\subseteq f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right)$. Hence, $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]$ $=\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)$ if f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$homeomorphism.

Conversely, if $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]=\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)$ for every subset A of $\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)$, then f is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$continuous. If A is $\mathcal{N}_{\mathrm{ano}}-$closed in $\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)$, then A is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in $\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)$. Then $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})=\mathrm{A}$ which implies $f\left[\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}\text{-}Cl(\mathrm{A})\right]=f(\mathrm{A})$. Hence, by the given hypothesis, it follows that $\mathcal{N}_{\mathrm{ano}}\text{-}Cl\left(f(\mathrm{A})\right)=f(\mathrm{A})$. Thus $f(\mathrm{A})$ is $\mathcal{N}_{\mathrm{ano}}-$closed in V and hence $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed in V for every $\mathcal{N}_{\mathrm{ano}}-$closed set A in U. That is, f is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed map. Therefore $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ is $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$homeomorphism.

7 Nano Quasi Ideal Ideal Semi $-\alpha-$Generalized $-$Open and Closed Mappings

Definition 7.1. A function $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ is said to be nano ideal quasi semi-α-generalized open $\left(\textit{briefly},\mathcal{N}_{ano}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}\textit{-open}\right)$, If the image of every $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in U is $\mathcal{N}_{\mathrm{ano}}-$open in U.

Theorem 7.2. A function $f:\left(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I}\right)\rightarrow\left(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J}\right)$ is $\mathcal{N}_{ano}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}\textit{-open}$ if and only if for every subset G of U, $f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})\right)\subseteq\mathcal{N}_{ano}-\mathrm{I}nt\left(f(\mathrm{G})\right)$.

Proof. Let f be a $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open function. Now, we have $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})\subseteq\mathrm{G}$ and $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})$ is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set. Hence, we obtain that $f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})\right)\subseteq f(\mathrm{G})$. As $f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})\right)$ is $\mathcal{N}_{\mathrm{ano}}-$open, $f\left(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G})\right)\subseteq\mathcal{N}_{ano}-\mathrm{I}nt\left(f(\mathrm{G})\right)$.

Conversely, assume that G is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in U, then, $f(\mathrm{G})=f(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G}))\subseteq\mathcal{N}_{ano}-\mathrm{I}nt(f(\mathrm{G}))$ but $\mathcal{N}_{ano}-\mathrm{I}nt(f(\mathrm{G}))\subseteq f(\mathrm{G})$. Consequently, $f(\mathrm{G})=\mathcal{N}_{ano}-\mathrm{I}nt(f(\mathrm{G}))$ and hence f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open.

Theorem 7.3. If a function $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open, then $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(f^{-1}(\mathrm{G}))\subseteq f^{-1}(\mathcal{N}_{ano}-\mathrm{I}nt(\mathrm{G}))$ for every subset G of V.

Proof. Let G be any arbitrary subset of V. Then, $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(f^{-1}(\mathrm{G}))$ is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in U and f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open, then $f(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(f^{-1}(\mathrm{G})))\subseteq\mathcal{N}_{ano}-\mathrm{I}nt(f(f^{-1}(\mathrm{G}))).\subseteq\mathcal{N}_{ano}-\mathrm{I}nt(\mathrm{G})$. Thus, $\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(f^{-1}(\mathrm{G}))\subseteq f^{-1}(\mathcal{N}_{ano}-\mathrm{I}nt(\mathrm{G}))$.

Definition 7.4. A subset G is said to be an $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$nrighbourhood of a point p of U if there exists a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set O such that $p\in\mathrm{O}\subseteq\mathrm{G}$.

Theorem 7.5. For a function $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$, the following are equivalent

(i) f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open;

(ii) for each subset G of U, $f(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{G}))\subseteq\mathcal{N}_{ano}-\mathrm{I}nt(f(\mathrm{G}))$;

(iii) for each $p\in\mathrm{U}$ and each $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$nrighbourhood G of p in U, there exists a $\mathcal{N}_{\mathrm{ano}}-$nrighbourhood H of $f(p)$ in V such that $\mathrm{H}\subseteq f(G)$.

Proof. (i) ⇒ (ii) It follows from Theorem 7.2.

(ii) ⇒ (iii) Let $p\in\mathrm{U}$ and G be an arbitrary $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$neighbouhood of $p\in\mathrm{U}$. Then, there exists a $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open set H in U such that $p\in\mathrm{H}\subseteq\mathrm{G}$ Then by (ii), we have $f(\mathrm{H})=f(\mathcal{N}_{ano}\,\mathrm{IS}\alpha\mathrm{g}-\mathrm{I}nt(\mathrm{H}))\subseteq\mathcal{N}_{ano}\,\mathrm{I}nt(f(\mathrm{H}))$ and hence $f(\mathrm{H})$ is $\mathcal{N}_{\mathrm{ano}}-$open in V such that $f(p)\in f(\mathrm{H})\subseteq f(\mathrm{G})$.

(iii) ⇒ (i) Let G be an arbitrary $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set in U. Then for each $p\in f(\mathrm{G})$, by (iii) there exists a $\mathcal{N}_{\mathrm{ano}}-$neighbourhood H_p of p in V such that $\mathrm{H}_p\subseteq f(\mathrm{G})$. As H_p is a $\mathcal{N}_{\mathrm{ano}}-$neighbourhood of p, there exists a $\mathcal{N}_{\mathrm{ano}}-$open set O_p in V such that $p\in\mathrm{O}_p\subseteq\mathrm{H}_p$. Thus $f(\mathrm{G})=\bigcup\{\mathrm{O}_p:p\in f(\mathrm{G})\}$ which is a $\mathcal{N}_{\mathrm{ano}}-$open set in V. This implies that f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open function.

Theorem 7.6. A function $f:(\mathrm{U},\mathrm{T}_{\mathrm{R}}(\mathrm{X}),\mathrm{I})\rightarrow(\mathrm{V},\sigma_{\mathrm{R}'}(\mathrm{Y}),\mathrm{J})$ is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open if and only if for any subset B of V and for any $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$closed set F of U containing $f^{-1}(\mathrm{B})$, there exists a $\mathcal{N}_{\mathrm{ano}}-$closed set G of V containing B such that $f^{-1}(\mathrm{G})\subseteq\mathrm{F}$.

Proof. Suppose f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open. Let $\mathrm{B}\subseteq\mathrm{V}$ and F be a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set of U containing $f^{-1}(\mathrm{B})$. Now, put $\mathrm{G}=\mathrm{V}-f(\mathrm{U}-\mathrm{F})$. It is clear that $f^{-1}(\mathrm{B})\subseteq\mathrm{F}$ implies $\mathrm{B}\subseteq\mathrm{G}$. Since f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open, we obtain G as a $\mathcal{N}_{\mathrm{ano}}-$closed set of V. Moreover, we have $f^{-1}(\mathrm{G})\subseteq\mathrm{F}$. Conversely, let G be a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$open set of U and put $\mathrm{B}=\mathrm{V}-f(\mathrm{G})$. Then $\mathrm{U}-\mathrm{G}$ is a $\mathcal{N}_{\mathrm{ano}}\,\mathrm{IS}\alpha\mathrm{g}-$closed set in U containing $f^{-1}(\mathrm{B})$. By hypothesis, there exists a $\mathcal{N}_{\mathrm{ano}}-$closed set F of V such that $\mathrm{B}\subseteq\mathrm{F}$ and $f^{-1}(\mathrm{F})\subseteq\mathrm{U}-\mathrm{G}$. Hence, we obtain $f(\mathrm{G})\subseteq\mathrm{V}-\mathrm{F}$. On the other hand, it follows that $\mathrm{B}\subseteq\mathrm{F}$, $\mathrm{V}-\mathrm{F}\subseteq\mathrm{V}-\mathrm{B}=f(\mathrm{G})$. Thus we obtain $f(\mathrm{G})=\mathrm{V}-\mathrm{F}$ which is $\mathcal{N}_{\mathrm{ano}}-$open and hence f is $\mathcal{N}_{\mathrm{ano}}\mathrm{Q}\,\mathrm{IS}\alpha\mathrm{g}-$open function.

Theorem 7.7. A function $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}Q\ IS\alpha g$-open if and only if $f^{-1}(\mathcal{N}_{ano}\text{-}Cl(B))\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f^{-1}(B))$ for every subset B of V.

Proof. Suppose that f is $\mathcal{N}_{ano}Q\ IS\alpha g$-open For any subset B of V, $f^{-1}(B)\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f^{-1}(B))$. Therefore, by Theorem 7.5, there exists a $\mathcal{N}_{ano}$-closed set F in V such that $B\subseteq F$ and $f^{-1}(F)\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(\mathcal{N}_{ano}\text{-}Cl(B))\subseteq f^{-1}(F)\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f^{-1}(B))$.

Conversely, let $B\subseteq V$ and F be a $\mathcal{N}_{ano}IS\alpha g$-closed set of U containing $f^{-1}(B)$. Put $H=\mathcal{N}_{ano}\text{-}Cl(B)$, then we have $B\subseteq W$ and W is $\mathcal{N}_{ano}$-closed and $f^{-1}(W)\subseteq\mathcal{N}_{ano}IS\alpha g\text{-}Cl(f^{-1}(B))\subseteq F$. Then by Theorem 7.6, f is $\mathcal{N}_{ano}Q\ IS\alpha g$-open.

Theorem 7.8. Consider two functions $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ and $g:(V,\sigma_{R'}(Y),J)\to(W,\gamma_{R''}(Z),L)$ and suppose $gof:(V,\sigma_{R'}(Y),J)\to(W,\gamma_{R''}(Z),L)$ is $\mathcal{N}_{ano}Q\ IS\alpha g$-open. If g is $\mathcal{N}_{ano}$-continuous injective function, then f is $\mathcal{N}_{ano}Q\ IS\alpha g$-open.

Proof. Let G be a $\mathcal{N}_{ano}Q\ IS\alpha g$-open set in U, then $(gof)(G)$ is $\mathcal{N}_{ano}$-open in W, since gof is $\mathcal{N}_{ano}Q\ IS\alpha g$-open. Again g is an $\mathcal{N}_{ano}$-continuous function, $f(G)=g^{-1}(gof(G))$ is $\mathcal{N}_{ano}$-open in V. This shows that f is $\mathcal{N}_{ano}Q\ IS\alpha g$-open.

Definition 7.9. A function $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is said to be Nano quasi-ideal semi-α generalized closed (briefly, $\mathcal{N}_{ano}Q\ IS\alpha g$-closed) if the image of every $\mathcal{N}_{ano}Q\ IS\alpha g$-closed set in U is $\mathcal{N}_{ano}$-closed in V.

Theorem 7.10. Every $\mathcal{N}_{ano}Q\ IS\alpha g$-closed function is $\mathcal{N}_{ano}$-closed as well as $\mathcal{N}_{ano}Q\ IS\alpha g$-closed.

Proof. It is obvious.

Theorem 7.11. A function $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}Q\ IS\alpha g$-closed if and only if for any subset B of V and for any $\mathcal{N}_{ano}IS\alpha g$-open set G of U containing $f^{-1}(B)$, there exists a $\mathcal{N}_{ano}$-open set H of V containing B such that $f^{-1}(H)\subseteq G$.

Proof. This proof is similar to that of theorem 7.6.

Theorem 7.12. Suppose that $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),L)$ be any two $\mathcal{N}_{ano}Q\ IS\alpha g$-closed functions. Then $gof:(U,T_R(X),I)\to(W,\Omega_{R''}(Z),L)$ is $\mathcal{N}_{ano}Q\ IS\alpha g$-closed function.

Proof. It is obvious.

Theorem 7.13. Suppose that $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g$-closed function and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),L)$ is $\mathcal{N}_{ano}Q\ IS\alpha g$-closed function. Then $gof:(U,T_R(X),I)\to(W,\Omega_{R''}(Z),L)$ is $\mathcal{N}_{ano}$-closed function.

Proof. It is obvious.

Theorem 7.14. Suppose that $f:(U,T_R(X),I)\to(V,\sigma_{R'}(Y),J)$ is $\mathcal{N}_{ano}IS\alpha g$-irresolute surjective function and $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),L)$ is a function such that $gof:(U,T_R(X),I)\to(W,\Omega_{R''}(Z),L)$ is a $\mathcal{N}_{ano}Q\ IS\alpha g$-closed function. Then $g:(V,\sigma_{R'}(Y),J)\to(W,\Omega_{R''}(Z),L)$ is a $\mathcal{N}_{ano}$-closed function.

Proof. Suppose that F is an arbitrary $\mathcal{N}_{ano}$ –closed set in V. Then F is $\mathcal{N}_{ano}\, IS\alpha g$–closed in V. As f is $\mathcal{N}_{ano}\, IS\alpha g$–irresolute, $f^{-1}(F)$ is $\mathcal{N}_{ano}\, IS\alpha g$–closed in U. Since $g o f$ is $\mathcal{N}_{ano} Q\, IS\alpha g$–closed and f is surjective, $(g o f)(f^{-1}(F)) = g(F)$, which is $\mathcal{N}_{ano}$ –closed in Z. This implies that g is a $\mathcal{N}_{ano}$ –closed function.

Theorem 7.15. A function $f:(U, T_R(X), I) \rightarrow (V, \sigma_{R'}(Y), J)$ is $\mathcal{N}_{ano} Q\, IS\alpha g$–closed if and only if $f(U)$ is $\mathcal{N}_{ano}$ –closed in V and $f(G) - f(U - G)$ is $\mathcal{N}_{ano}$ –open in $f(U)$ whenever G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U.

Proof. **Necessity:** Suppose $f:(U, T_R(X), I) \rightarrow (V, \sigma_{R'}(Y), J)$ is a $\mathcal{N}_{ano} Q\, IS\alpha g$–closed function. Since U is $\mathcal{N}_{ano}\, IS\alpha g$–closed, $f(U)$ is $\mathcal{N}_{ano}$ –closed in V and $f(G) - f(U - G) = f(G) \cap f(U) - f(U - G)$ is $\mathcal{N}_{ano}$ –open in $f(U)$ when G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U.

Sufficiency: Suppose $f(U))$ is $\mathcal{N}_{ano}$ –closed in V, $f(G) - f(U - G)$ is $\mathcal{N}_{ano}$ –open in $f(U)$ when G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U and let H be $\mathcal{N}_{ano}$ –closed in U. Then $f(H) = f(U) - (f(U - H) - f(H))$ is $\mathcal{N}_{ano}$ –closed in $f(U)$ and hence $\mathcal{N}_{ano}$ –closed in V,

Corollary 7.16. A surjective function $f:(U, T_R(X), I) \rightarrow (V, \sigma_{R'}(Y), J)$ is $\mathcal{N}_{ano} Q\, IS\alpha g$–closed if and only if $f(G) - f(U - G)$ is $\mathcal{N}_{ano}$ –open in V whenever G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U.

Proof. It is obvious.

Theorem 7.17. Let $f:(U, T_R(X), I) \rightarrow (V, \sigma_{R'}(Y), J)$ be $\mathcal{N}_{ano}\, IS\alpha g$–continuous and $\mathcal{N}_{ano} Q\, IS\alpha g$–closed surjective function. Then the topology on V is $\{f(H) - f(U - H) : H \textit{ is } \mathcal{N}_{ano}\, IS\alpha g\textit{-open in } U\}$.

Proof. Let M be $\mathcal{N}_{ano}$ –open in V. Let $f^{-1}(M)$ is $\mathcal{N}_{ano}\, IS\alpha g$–open in U, and $f(f^{-1}(M)) - f(U - f^{-1}(M)) = M$. Hence all $\mathcal{N}_{ano}$ –open sets in V are of the form $f(G) - f(U - G)$, G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U. On the other hand, all sets of the form $f(G) - f(U - G)$, G is $\mathcal{N}_{ano}\, IS\alpha g$–open in U, are $\mathcal{N}_{ano}$ –open in V from Corollary 7.16.

Definition 7.18. A $\mathcal{N}_{ano}$ –ideal –Top. –Space $(U, T_R(X), I)$ is said to be $\mathcal{N}_{ano}$ –normal if for any pair of disjoint $\mathcal{N}_{ano}$ –closed subsets E and F of U there exists disjoint $\mathcal{N}_{ano}$ –open sets G and H such that $E \subseteq G$ and $F \subseteq H$.

Definition 7.19. A $\mathcal{N}_{ano}$ –ideal –Top. –Space $(U, T_R(X), I)$ is said to be $\mathcal{N}_{ano}\, IS\alpha g$–normal if for any pair of disjoint $\mathcal{N}_{ano}\, IS\alpha g$–closed subsets E and F of U there exists disjoint $\mathcal{N}_{ano}$ –open sets G and H such that $E \subseteq G$ and $F \subseteq H$.

Theorem 7.20. Let $f:(U, T_R(X), I) \rightarrow (V, \sigma_{R'}(Y), J)$ be a $\mathcal{N}_{ano}\, IS\alpha g$–continuous and $\mathcal{N}_{ano} Q\, IS\alpha g$–closed surjective function with U is $\mathcal{N}_{ano}\, IS\alpha g$–normal. Then V is $\mathcal{N}_{ano}$ –normal.

Proof. Let K and L be disjoint $\mathcal{N}_{ano}$ –closed subsets of V. Then $f^{-1}(K)$, $f^{-1}(L)$ are disjoint $\mathcal{N}_{ano}\, IS\alpha g$–closed subsets of U. Since U is $\mathcal{N}_{ano}\, IS\alpha g$–normal, there exists disjoint $\mathcal{N}_{ano}$ –open sets G and H such that $f^{-1}(K) \subseteq G, f^{-1}(L) \subseteq H$. Then $K \subseteq f(G) - f(U - G)$ and $L \subseteq f(H) - f(U - H)$, further by corollary 7.16, $f(G) - f(U - G)$ and $f(H) - f(U - H)$ are $\mathcal{N}_{ano}$ –open sets in V and clearly $[f(G) - f(U - G)] \cap [f(H) - f(U - H)] = 0_N$. This shows that V is $\mathcal{N}_{ano}$ –normal.

Conclusions

The author has introduced the notions of new classes of functions called $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$continuous maps, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$irresolute-maps, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$open-maps, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$closed-maps, strongly $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$open-maps, strongly $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$closed-maps and $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$homeomorphisms, $\mathcal{N}_{ano}\mathtt{Q}\,\mathtt{IS}\alpha\mathtt{g}-$open-maps, and $\mathcal{N}_{ano}\mathtt{Q}\,\mathtt{IS}\alpha\mathtt{g}-$closed-maps and has investigated fundamental properties and characterizations of these mappings in $\mathcal{N}_{ano}-$Ideal$-$Top$-$Spaces. In future work, the author intends to continue to introduce and investigate properties and characterizations of the notions of $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$separation Axioms namely $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-T_0$ space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-T_1$ space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-T_2$ space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$regular space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$normal space, almost $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$normal space, mildly $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$normal space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$compact space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$Lindelof space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$connected space, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$closed graph and strongly $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$closed graph in $\mathcal{N}_{ano}-$Ideal$-$Top$-$Spaces. In future work, $\mathcal{N}_{ano}\,\mathtt{IS}\alpha\mathtt{g}-$open sets can be applied in an application field of real-life experience.

Scientific Ethics Declaration

* The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the author.

Conflict of Interest

* The author declares that he has no conflict of Interest.

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