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Evaluation of Nonlinear Fracture Quantities in Lime-Pumice Mixtures

Erkin Eren
Firat University

Ragip Ince
Firat University

Abstract: The tests on fracture mechanics of quasi-brittle materials until the 1970s indicated that linear elastic fracture mechanics (LEFM) was not valid for cement-based materials such as mortar and concrete. This inapplicability of LEFM is due to the existence of an inelastic zone, which is also so-called fracture process. Therefore, numerous non-Hookean fracture models have been introduced to evaluate fracture quantities in quasi-brittle materials. According to the two-parameter model (TPM) in concrete fracture, a free crack propagates unstably whenever the stress intensity factor and the crack tip opening displacement are equal to their threshold values, such as Mode I fracture toughness and critical crack tip opening displacement, respectively. In this study, beams, cubes and square prismatic specimens with three different initial crack lengths were produced to calculate fracture quantities of lime-pumice mixtures. Beams with a maximum aggregate diameter (g) of 4 mm were kept in the mold for 28 days while cubes with g=4 mm and square prismatic specimens with g=16 mm were saved in the mold for 90 days. The fracture quantities of the mixtures were evaluated by employing the modified peak load method, which was recently derived to determine fracture properties of TPM.

Keywords: Fracture mechanics, Lime, Pumice, Two-parameter model

Introduction

The first attempt to apply Linear Elastic Fracture Mechanics (LEFM) to cement-based composites was conducted by Kaplan (1961) and later expanded upon by Kesler et al. (1972). Their work revealed that LEFM does not adequately capture the fracture response of concrete. This limitation stems from the presence of a relatively large inelastic zone ahead of the crack tip in quasi-brittle materials such as concrete mixtures, rocks, and asphalt concrete. To address this shortcoming, several nonlinear fracture mechanics models have been introduced to more realistically describe fracture-governed failures in concrete. Among these are the fictitious crack model of Hillerborg et al. (1976), the crack band model proposed by Bazant and Oh (1983), the two-parameter model (TPM) by Jenq and Shah (1985), the effective crack model of Nallathambi and Karihaloo (1986), the size effect model (SEM) introduced by Bazant and Kazemi (1990), the double-K model developed by Xu and Reinhardt (1999), and the boundary effect method by Hu and Duan (2008).

Roman mortars represent the earliest form of concrete, composed of lime, water, sand, and volcanic tuff, facilitating the creation of edifices that have endured for almost two thousand years, with notable instances including the Pantheon temple and the Colosseum. The Pantheon temple continues to function as an exhibition hall today. The Colosseum suffered significant damage due to a catastrophic earthquake in 412 AD. Previously, during the period when Rome embraced Christianity, numerous valuable stone fragments were incorporated into the construction of various other buildings. Currently, one-third of the Colosseum remains, predominantly consisting of its concrete structure. Conversely, in Seljuk and Ottoman architectural styles, a substantial number of buildings constructed with Khorasan mortar-comprising lime, sand, water, and ground-baked clay (brick) have persisted to the present day.

In this research, beams, cubes, and square prismatic specimens featuring three distinct initial crack lengths were created to assess the fracture characteristics of lime-pumice mixtures. Beams with a maximum aggregate diameter (g) of 4 mm were maintained in the mold for a duration of 28 days, whereas cubes with g=4 mm and square prismatic specimens with g=16 mm were retained in the mold for 90 days. The fracture characteristics of the mixtures were analyzed using the modified peak load method, which has been recently developed to evaluate the fracture properties of TPM.

The Two-Parameter Model (TPM) in Concrete Fracture

During the 1970s, experimental investigations into the fracture behavior of cement-based composites—including paste, mortar, and concrete—indicated that the traditional framework of Linear Elastic Fracture Mechanics (LEFM) could not adequately represent quasi-brittle materials (Kesler et al., 1972). The primary limitation originates from the formation of a relatively large inelastic region, referred to as the fracture process zone (FPZ), which develops in front of and surrounding major crack tips. To address this issue, several nonlinear fracture mechanics models have been introduced to more accurately describe the FPZ.

These approaches are generally classified into cohesive crack models and effective crack models, one notable example being the two-parameter fracture model (TPM) proposed by Jenq and Shah (1985). The fundamental objective of these models is to evaluate the critical crack extension, expressed as $\Delta a = a_c - a_0$, where ac denotes the crack length at peak load and a_0 represents the initial notch length. This parameter is typically obtained from either the load–displacement curve or the load–crack mouth opening displacement relationship of the tested structure.

According to the Two-Parameter Model (TPM), fracture in a concrete structure occurs when the stress intensity factor (K_I) and the crack tip opening displacement (*CTOD*) both attain their corresponding critical thresholds, denoted as K^s_{Ic} and $CTOD_c$. These characteristic fracture parameters can be evaluated using the following formulations derived from Linear Elastic Fracture Mechanics (LEFM):

$$K_{Ic}^{S} = \sigma_{Nc}\sqrt{\pi a_c}\,Y(g,l) \qquad (1)$$

$$CTOD_c = \frac{\gamma \sigma_{Nc} a_c}{E} V_1(g,l) M(g,l) \qquad (2)$$

Here, σ_{Nc} denotes the nominal failure stress, E represents Young's modulus, and Y, V_1, and M are dimensionless parameters that are functions of both the structural geometry (g) and the loading configuration (l). In Eq. (2), the coefficient γ takes the value of π for splitting specimens and 4 for beams. The term Y is commonly referred to as the geometry factor, while M is obtained from the ratio COD(a_c)/CMOD$_c$, where $CMOD_c$ corresponds to the critical crack mouth opening displacement. The TPM is particularly convenient for structural applications, since the functions Y, V_1, and M can be readily found in Linear Elastic Fracture Mechanics (LEFM) reference handbooks (Tada et al., 2000).

In the two-parameter model (TPM), fracture properties can be evaluated using two main experimental procedures: the compliance method, first proposed by Jenq and Shah (1985), and the peak-load method, later developed by Tang et al. (1996). The compliance approach determines the parameters from the correlation between the applied load and the crack mouth opening displacement (P–CMOD) obtained from a three-point bending specimen of width *b*, depth *d*, and span *S*. This procedure requires a closed-loop testing system, as schematically shown in Figure 1a. Within TPM, the critical crack length (a_c) is calculated from the unloading compliance (C_u) measured at 95% of the maximum load, as illustrated in Figure 1b.

The peak-load method, in contrast, eliminates the need for sophisticated closed-loop equipment and thus provides a simpler alternative to the RILEM compliance-based procedure for calculating fracture parameters in TPM. Nevertheless, this approach still requires a minimum of three specimens to account for the variability inherent in concrete. The test specimens may either share identical geometrical dimensions but differ in their initial notch lengths, or they may exhibit consistent notch sizes while varying in overall dimensions. For each individual specimen, the governing equations of TPM can be expressed as formulated by Tang et al. (1992).

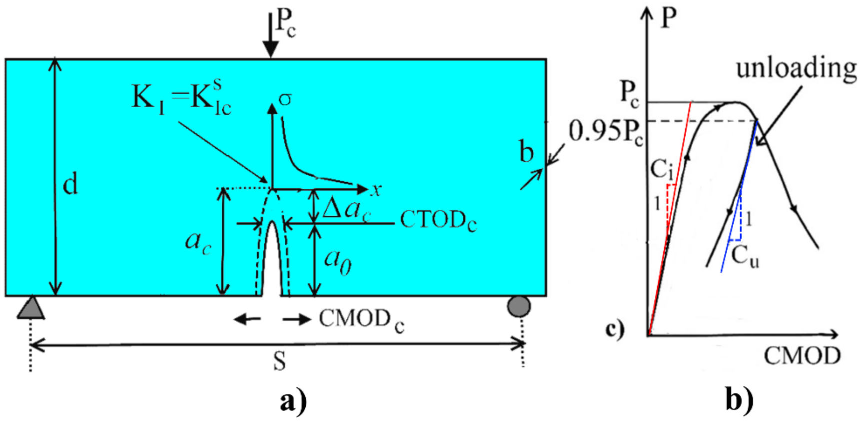

Figure 1. Modeling based on TPM a) notched beam b) typical load-CMOD curve

$$K_I^i\left(\sigma_{Nc}^i, a_c^i\right) = K_{Ic}^S \qquad , i = 1,2$$
$$CTOD^i\left(\sigma_{Nc}^i, a_c^i\right) = CTOD_c \tag{3}$$

here i denotes the ith specimen. Consequently, the fracture parameters can be found by simultaneously solving four non-linear equations. However, three or more distinct specimens must be tested to ensure statistically valid results because random errors always exist in measured values of σ^1_{Nc} and σ^2_{Nc}.

Ince (2025a) was recently developed the modified peak load method based on an optimization procedure to calculate K^s_{Ic} and $CTOD_c$. This approach is based on the simultaneous solution of equation (3) as the failure criterion of a concrete structure. As mentioned above, the main aim of any fracture model is to evaluate the critical crack extension (Δa) at the peak load. Accordingly, for this problem, it may be sufficient to find the nominal strength value (σ_{Nc}) corresponding to the peak load and the Δa value for the initial crack length (a_0) for each sample. These Δa values calculated for each sample should be such that the same fracture parameters (K^s_{Ic} and $CTOD_c$) are met for all tested samples. However, it is impossible to obtain an exact solution for a heterogeneous material like concrete. Consequently, such a problem can only be approached using optimization techniques. To find the fracture parameters of TPM, the following two expressions were initially minimized by utilizing the least squares error criterion:

$$f\left(K_{Ic}^S\right) = \sum_{i=1}^{n} \Delta^2\left(K_{Ic}^S\right) = \sum_{i=1}^{n}\left(K_{Ic,i}^S - \overline{K_{Ic,i}^S}\right)^2 \tag{4}$$

$$f\left(CTOD_c\right) = \sum_{i=1}^{n} \Delta^2\left(CTOD_c\right) = \sum_{i=1}^{n}\left(CTOD_{c,i} - \overline{CTOD_{c,i}}\right)^2 \tag{5}$$

Here n is the number of samples tested, $\overline{K_{Ic,i}^S}$ is the average value of K^s_{Ic} and $K^s_{Ic,i}$ is the value of K^s_{Ic} or the i^{th} sample in Equation 4 while $\overline{CTOD_{c,i}}$ is the average value of $CTOD_c$ and $CTOD_{ci}$ is the value of $CTOD_c$ for the i^{th} sample in Equation 5. On the other hand, to provide the simultaneous solution of the above two minimization equations, the root sum squared (RSS) method, which is also called the statistical tolerance analysis method, was used as follows:

$$RSS = \sqrt{\left(f\left(K_{Ic}^S\right)\right)^2 + \left(f\left(CTOD_c\right)\right)^2} \tag{6}$$

Note that although, in practice, K^s_{Ic} and $CTOD_c$ are commonly used in terms of MPa√m and mm, respectively, it is recommended to choose MPa√mm and µm because the quantities in these units are very different from each other. The procedures described above, which form the basis of the modified peak load method, can be easily implemented using a spreadsheet-based program such as the MS-EXCEL-based SOLVER toolkit.

Fracture test specimens such as beams (Figure 1a), notched split tension cylinders/cubes (Figure 2a), compact compression specimens (Figure 2b), wedge-splitting specimens (Figure 2c), cube with edge notch (Figure 2d) and semi-circular bending (SCB) specimens (Figure 2e) can be employed in the peak-load method, only beams are initially used in the compliance method proposed by RILEM. However, the SCB specimens were recently used to analyze rock materials by Ince (2025b) according to the compliance method.

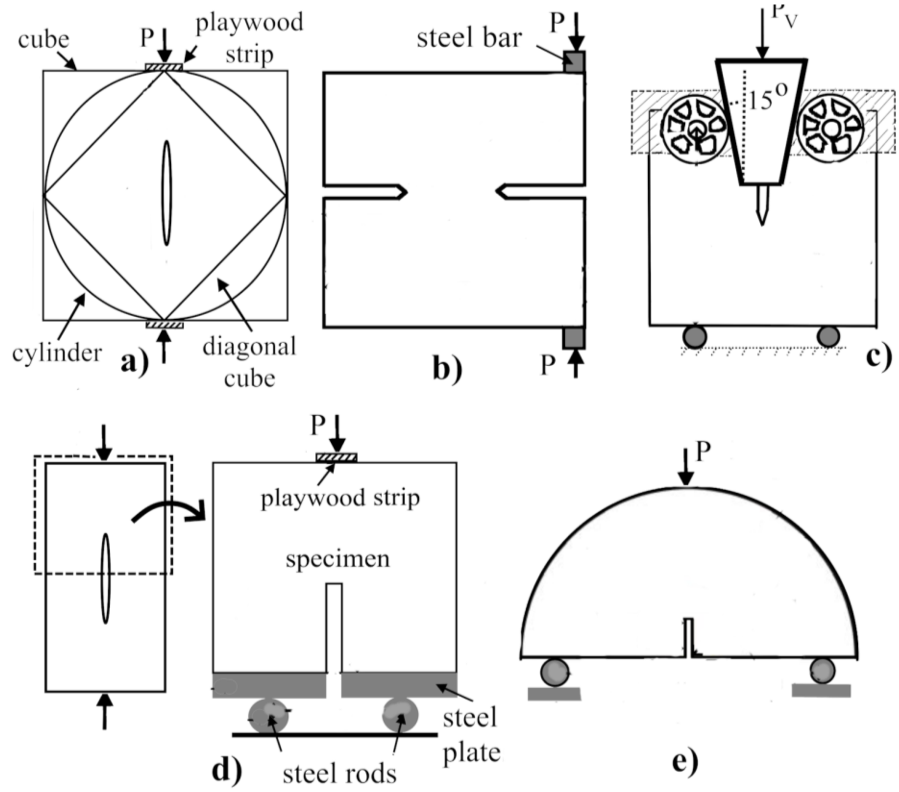

Figure 2. Compact specimens used in concrete fracture a) splitting specimens b) compact compression specimens c) wedge spitting specimens d) cube with edge notch e) semi-circular bending specimens

Experimental Program

The mixtures with a maximum aggregate diameter of 4 mm were in a water: lime: pumice: aggregate ratio of 1.53:1.00:1.00:6.00, while the mixture with a maximum aggregate diameter of 16 mm were in a water: lime: pumice: aggregate ratio of 1.42:1.00:1.00:6.00. CL-80-S lime with the specific gravity=2.08 was used in the mixture with beams while CL-90-S lime with the specific gravity=2.09 was employed in the other mixtures. The aggregates were air-dried prior to mixing. The Fuller parabola was used to determine the percentages of the aggregate gradation.

The widths of all the beams with a maximum aggregate diameter of 4 mm were taken as constant, b =50 mm. The lengths of beams were taken as L=3d (Figure 1a). The cubes with 100 mm were used for the mixtures with a maximum aggregate diameter of 4 mm. The square prisms were designed as 100×150×150 mm for mixtures with a maximum aggregate diameter of 16 mm. All beams were kept in the mold for 28 days, while the cubes and the square prisms were saved in the mold for 90 days. All specimens were wrapped in stretch film. The specimens were notched with a diamond saw blade of 1.5 mm thickness on the day of the fracture test. In addition to the notched specimens, cubes (100 mm) were made to determine compressive strength. While beams of the same dimensions were used to determine the flexural strength of the beams, 100 mm cubes were made to determine the splitting strength of the other specimen configurations.

The compression tests, the splitting tests and the bending tests were performed using a digital compression machine with a capacity of 100 kN. The specimens were loaded monotonically until final failure and care was taken to apply a constant loading rate (Figure 3). Typically, approximately 2 min (± 30 sec) elapsed before the maximum load capacity for each specimen was reached.

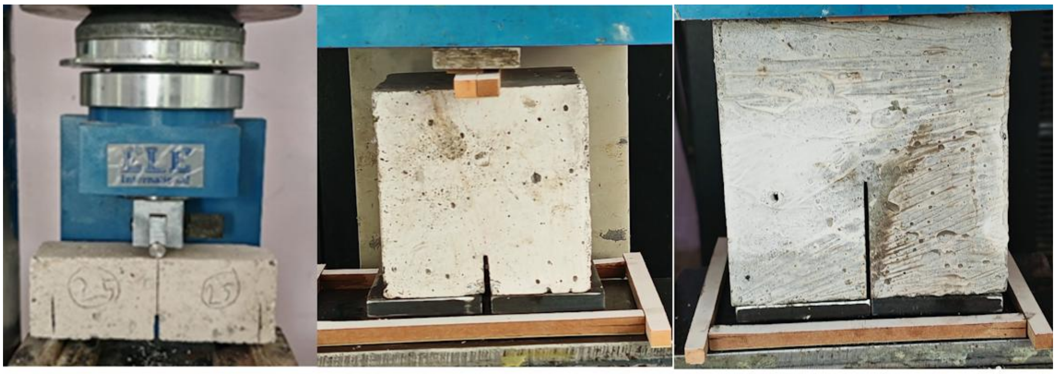

Figure 3. Test setup of the notched specimens

In this study, by using the following formulas, the dynamic Young's moduli of the materials were determined by means of the ultrasonic technique for prismatic specimens and cubical specimens, respectively:

$$E_d = 10^3 V^2 \Delta, \quad \text{MPa} \tag{7}$$

$$E_d = 10^3 \frac{V^2 \Delta (1+\nu)(1-2\nu)}{1-\nu}, \quad \text{MPa} \tag{8}$$

where, V is the velocity (km/s) and Δ is the unit weight (kg/lt). Since lime-pumice mixtures are low strength materials, the Poisson ratio (ν) was taken as 0.3 in this study (Erdogan, 2003). Note that the dynamic Young's moduli of materials (E_d) are larger than static Young's moduli (E_s). The formula: E_s=6/7×E_d was used for lime-pumice mixtures in this study (Lydon, 1972). Physical and mechanical properties of lime-pumice mixtures used in this study are summarized in Table 1.

The specimen width (b), specimen depth (d) and the peak load values (P_c) of the notched beams are reported in Table 2 according to the notch depths (a_0). The values of a_0 and P_c of the splitting specimens: cubes and square prisms are summarized in Table 3. In these tables, the letters B, C, and SP refer to beams, cubes, and square prism specimens.

Crack patterns at the failure of beams and compression cubes are shown in Figure 4. The specimen width (b), specimen depth (d) and the peak load values (P_c) of the notched beams were reported in Tables 1 according to the notch depths (a_0).

Applications of the Modified Peak Load Method to Lime-Pumice Mixtures

Tables 4, 5, and 6 illustrate the applications of the modified peak load method to mixtures in this study. The test data are summarized for each sample in the first two columns of these tables. In these tables, the nominal strengths were computed as σ_{Nc}=3.75P/(b×d) and σ_{Nc}=P/(π×b×d) for beams and splitting specimens, respectively. For each sample, the crack extensions (Δa), which were taken as variables of the optimization problem, were initially selected as 5 mm. The normalized functions of the cracked structure, such as Y and V_1, were also summarized while the function of M in Equation 2 is presented as embedded in $CTOD_c$. All analysis was conducted by using the MS-EXCEL-based SOLVER toolkit. The root sum squared (RSS) values in these tables are the minimized values.

Table 1. Physical and mechanical properties of lime-pumice mixtures used in this study

Mix	g mm	w/b	Δ g/cm^3	T Days	f_c MPa	f_t MPa	f_{sp} MPa	E_s MPa
M4-28	4	0.77	1.93	28	3.79	1.181	-	13384
M4-90	4	0.77	2.01	90	9.82	-	1.300	14216
M16-90	16	0.71	2.09	90	10.82	-	1.305	20382

Table 2. Test results of beams tested in this study

Specimen	b mm	d mm	a_0 mm	P_c N
B1	47.77	49.32	8.84	540
B2	50.85	50.3	9.99	460
B3	54.16	49.63	9.71	590
B4	50.41	50.23	9.36	680
B5	52.08	50.41	9.93	520
B6	53.65	50.03	15.00	350
B7	54.26	49.84	14.26	490
B8	49.26	50.05	13.58	410
B9	52.21	51.28	15.79	460
B10	52.32	50.72	15.42	450
B11	48.02	49.63	23.12	290
B12	50.32	49.17	21.84	250
B13	55.85	49.21	21.43	320

Table 3. Test results of cubes (C) and square spasmatic (SP) tested in this study

Specimen	a_0 mm	P_c kN	Specimen	a_0 mm	P_c kN
C1	29.35	17.65	SP1	27.89	50.28
C2	28.5	23.51	SP2	28.45	48.14
C3	29.52	16.44	SP3	28.37	48.81
C4	39.91	14.9	SP4	46.25	39.87
C5	39.32	12.56	SP5	47.86	41.10
C6	40.07	12.18	SP6	45.15	44.22
C7	47.38	11.61	SP7	65.37	27.70
C8	47.91	10.16	SP8	65.7	16.75
C9	48.57	9.02	SP9	65.66	18.03

The following formulas were applied to beams with a span/depth ratio of 2.5 in this study (Yang et al., 1997):

$$Y(\alpha) = \frac{1.83 - 1.65\alpha + 4.76\alpha^2 - 5.3\alpha^3 + 2.51\alpha^4}{\sqrt{\pi}(1+2\alpha)(1-\alpha)^{3/2}} \tag{9}$$

$$V_1(\alpha) = 0.65 - 1.88\alpha + 3.02\alpha^2 - 2.69\alpha^3 + \frac{0.68}{(1-\alpha)^2} \tag{10}$$

$$M(\alpha_0, \alpha_c) = \sqrt{\left(1 - \frac{\alpha_0}{\alpha_c}\right)^2 + (1.081 - 1.149\alpha_c)\left[\frac{\alpha_0}{\alpha_c} - \left(\frac{\alpha_0}{\alpha_c}\right)^2\right]} \tag{11}$$

For the cube and the square prismatic specimens subjected to splitting loading used in Table 2, the LEFM formulas are given by (Ince, 2021):

$$Y(\alpha) = 0.3691 + 0.4492\alpha + 1.3284\alpha^2 + 2.4467\alpha^3 \quad \text{for } \beta\text{=0.1} \tag{12}$$

$$Y(\alpha) = 0.3341 + 0.0101\alpha + 2.5609\alpha^2 + 0.4067\alpha^3 \quad \text{for } \beta\text{=0.2} \tag{13}$$

$$V_1(\alpha) = 0.3722 + 2.0219\alpha - 6.6125\alpha^2 + 14.3038\alpha^3 \quad \text{for } \beta\text{=0.1} \tag{14}$$

$$V_1(\alpha) = 0.3040 + 1.7717\alpha - 5.3749\alpha^2 + 12.2420\alpha^3 \quad \text{for } \beta\text{=0.2} \tag{15}$$

$$M\left(\alpha_0,\alpha_c\right)=\sqrt{\left(1-\frac{\alpha_0}{\alpha_c}\right)^2+\left(2.16-4.02\alpha_c\,(1-\beta)^{-0.23}\,\alpha_c^{3.67}\left(\frac{\alpha_0}{\alpha_c}\right)^{1.15}\right)\left[\frac{\alpha_0}{\alpha_c}-\left(\frac{\alpha_0}{\alpha_c}\right)^2\right]} \tag{16}$$

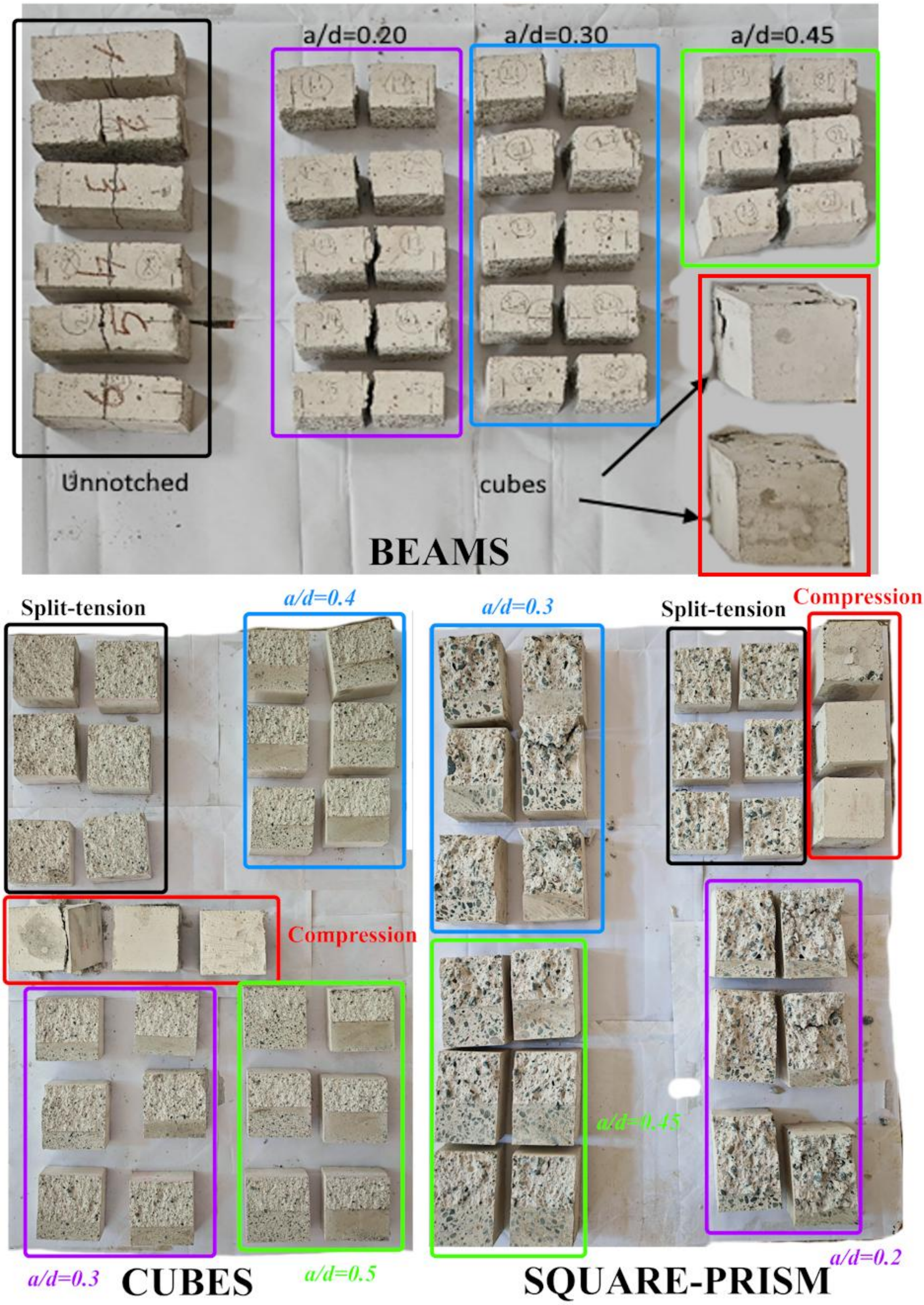

Figure 4. Fractured specimens

Table 4. Application of the modified peak load method to beams with g=4 mm

No	a_0 mm	σ_{Nc} MPa	Δa mm	a_c mm	a_c/d	Y	K^s_{Ic} MPa√mm	V_1	$CTOD_c$ μm		$\Delta^2(K^s_{Ic})$	$\Delta^2(CTOD_c)$
1	8.84	0.860	4.68	13.52	0.274	0.967	5.41	1.559	2.931		0.0012	0.0008
2	9.99	0.674	6.59	16.58	0.330	1.018	4.95	1.731	3.302		0.1807	0.1171
3	9.71	0.823	4.65	14.36	0.289	0.979	5.41	1.601	2.934		0.0010	0.0007
4	9.36	1.007	2.99	12.35	0.246	0.949	5.95	1.491	2.509		0.3260	0.2029
5	9.93	0.743	5.62	15.55	0.309	0.996	5.17	1.659	3.129		0.0435	0.0286
6	15.00	0.489	6.87	21.87	0.437	1.188	4.82	2.263	3.417		0.3170	0.2093
7	14.26	0.679	4.11	18.37	0.369	1.068	5.51	1.888	2.853		0.0173	0.0113
8	13.58	0.624	5.31	18.89	0.377	1.081	5.19	1.929	3.112		0.0348	0.0231
9	15.79	0.644	3.96	19.75	0.385	1.093	5.55	1.966	2.826		0.0283	0.0180
10	15.42	0.636	4.21	19.63	0.387	1.096	5.47	1.975	2.885		0.0086	0.0056
11	23.12	0.456	2.80	25.92	0.522	1.420	5.85	3.001	2.558		0.2182	0.1617
12	21.84	0.379	4.87	26.71	0.543	1.495	5.19	3.251	3.129		0.0361	0.0285
13	21.43	0.437	3.97	25.40	0.516	1.398	5.45	2.932	2.894		0.0056	0.0043
						Mean	5.38	Mean	2.96	Σ	1.2184	0.8119
											RSS	1.44641

Table 5. Application of the modified peak load method to cubes with g=4 mm

No	a_0 mm	σ_{Nc} MPa	Δa mm	a_c mm	a_c/d	Y	K^s_{Ic} MPa√mm	V_1	$CTOD_c$ μm		$\Delta^2(K^s_{Ic})$	$\Delta^2(CTOD_c)$
1	29.35	0.562	12.04	41.39	0.414	0.956	6.13	1.176	4.31		0.0258	0.0232
2	28.50	0.748	8.00	36.50	0.365	0.829	6.64	0.990	3.80		0.1277	0.1240
3	29.52	0.523	13.11	42.63	0.426	0.992	6.00	1.232	4.42		0.0797	0.0689
4	39.91	0.474	7.45	47.36	0.474	1.140	6.59	1.480	3.90		0.0934	0.0641
5	39.32	0.400	10.42	49.74	0.497	1.222	6.11	1.629	4.30		0.0321	0.0218
6	40.07	0.388	10.39	50.46	0.505	1.248	6.09	1.678	4.31		0.0373	0.0248
7	47.38	0.370	6.58	53.96	0.540	1.383	6.65	1.937	3.88		0.1341	0.0756
8	47.91	0.323	8.16	56.07	0.561	1.470	6.31	2.114	4.14		0.0005	0.0003
9	48.57	0.287	9.55	58.12	0.581	1.559	6.05	2.300	4.33		0.0561	0.0315
						Mean	6.29	Mean	4.16	Σ	0.5868	0.4343
											RSS	0.7300

Table 6. Application of the modified peak load method to square prisms with g=16 mm

No	a_0 mm	σ_{Nc} MPa	Δa mm	a_c mm	a_c/d	Y	K^s_{Ic} MPa√mm	V_1	$CTOD_c$ μm		$\Delta^2(K^s_{Ic})$	$\Delta^2(CTOD_c)$
1	27.89	1.067	18.42	46.31	0.309	0.593	7.63	0.699	4.35		0.0448	0.0726
2	28.45	1.022	19.13	47.58	0.317	0.608	7.59	0.716	4.40		0.0639	0.1013
3	28.37	1.036	18.82	47.19	0.315	0.603	7.61	0.711	4.38		0.0557	0.0888
4	46.25	0.846	10.37	56.62	0.377	0.725	8.18	0.865	3.74		0.1084	0.1170
5	47.86	0.872	8.62	56.48	0.377	0.723	8.40	0.863	3.53		0.3034	0.3053
6	45.15	0.938	8.72	53.87	0.359	0.687	8.39	0.814	3.52		0.2906	0.3109
7	65.37	0.588	6.79	72.16	0.481	0.977	8.64	1.275	3.45		0.6367	0.4024
8	65.70	0.356	18.14	83.84	0.559	1.211	6.99	1.753	4.76		0.7399	0.4566
9	65.66	0.383	16.33	81.99	0.547	1.171	7.19	1.666	4.61		0.4304	0.2757
						Mean	7.85	Mean	4.08	Σ	2.6738	2.1307
											RSS	3.4190

Conclusion

This study employed the modified peak load method based on the two-parameter model in concrete fracture to examine the fracture behavior of lime-pumice mixtures. The key findings are summarized below:

The fracture toughness value of the lime-pumice mortar is around 1/5 compared to normal cementitious mortars in 28 days test. Moreover, the resistance of this material to crack propagation is much lower than normal cement-based ones. The fracture toughness value of 90-day mortar is approximately 20% greater than that of 28-day mortar. On the other hand, the crack tip opening displacement value of 90-day mortar is approximately 30%

greater than that of 28-day mortar. The fracture toughness value of the lime-pumice mixture with the maximum aggregate size=16 mm is approximately 25% greater than that of the lime-pumice mixture with the maximum aggregate size=4 mm while no significant difference is observed between the crack tip opening displacement values.

Recommendations

Further studies can come up with more reliable results by investigating various types and sizes of aggregates to verify the above findings.

Scientific Ethics Declaration

* The authors declare that the scientific ethical and legal responsibility of this article published in EPSTEM journal belongs to the authors.

Conflict of Interest

* The authors declare that they have no conflicts of interest

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Author(s) Information

Erkin Eren
Firat University
Engineering Faculty, Elazig, Türkiye
Contact e-mail: *eeren@firat.edu.tr*

Ragip Ince
Firat University
Engineering Faculty, Elazig, Türkiye

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