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GCD Matrices Associated with Nathanson's Totient Function

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Abstract: Let $\Phi(n)$ denote Nathanson's Euler-type function for subsets of $\{1, 2, \dots, n\}$, defined as the number of nonempty subsets A satisfying $\gcd(\gcd A, n) = 1$. We introduce the second Möbius transform $\Psi = \mu * \Phi$, derive explicit formulas for Ψ , prove $\Psi(n) > 0$ for every positive integer n , and obtain lower bounds and asymptotic estimates. We then study the Nathanson GCD matrix $G_\Phi(S) = \left[\Phi(\gcd(x_i, x_j)) \right]_{i,j=1}^r$ associated with a finite set S of distinct positive integers. Using divisor-incidence factorization, we prove positive definiteness for arbitrary finite sets, derive exact determinant formulas, and obtain an explicit inverse formula on factor-closed sets.

Keywords: Nathanson's totient function; GCD matrix; Möbius inversion; Dirichlet convolution; Positive definite matrix; Factor-closed set; Mersenne numbers.

Introduction

Arithmetic functions and structured matrices provide a natural meeting point between number theory and linear algebra. One of the classical examples is the GCD matrix whose (i, j) -entry is $\gcd(x_i, x_j)$, defined on a finite set $S = \{x_1, \dots, x_r\}$ of distinct positive integers. The subject originates with Smith's classical arithmetical determinant (Smith, 1875–1876) and was developed systematically in the modern literature by Beslin and Ligh (1989). Subsequent work extended these ideas to matrices generated by general arithmetic functions and to the broader theory of meet and join matrices; see Korkee and Haukkanen (2003) and Ovall (2004). Divisor-incidence factorizations are particularly effective for studying determinants, inverses, nonsingularity, and positive definiteness of matrices whose entries have the form $f(\gcd(x_i, x_j))$.

Another direction in arithmetic combinatorics concerns analogues of Euler's totient function obtained by counting relatively prime subsets. Nathanson (2007) introduced a ϕ function for subsets of $\{1, 2, \dots, n\}$; in the notation adopted here, $\Phi(n)$ counts the nonempty subsets $A \subseteq \{1, \dots, n\}$ for which $\gcd(\gcd(A), n) = 1$. Nathanson obtained exact formulas for this function, while Nathanson and Orosz (2007) developed related asymptotic estimates. A fundamental identity is $\sum \Phi(d) = 2^n - 1$, where the sum is over all divisors d of n . Thus Φ is naturally connected with the Mersenne sequence $M(n) = 2^n - 1$. In the language of Dirichlet convolution, the preceding identity is $M = 1 * \Phi$, and Möbius inversion gives $\Phi = \mu * M$. Standard background on arithmetic functions, Möbius inversion, and Dirichlet convolution may be found in Apostol (1976) and McCarthy (1986). This convolutional relation is the starting point of the present investigation.

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Mersenne-type arithmetic matrices have also been studied recently. Awad, Mghames, and Amin (2024) investigated Mersenne power GCD matrices and obtained structural results for associated determinants and inverses. Zeid, Chehade, and Awad (2025) subsequently considered Mersenne GCED matrices in the setting of exponential divisors. These works motivate a closer examination of arithmetic functions arising from the Mersenne sequence through Möbius transformation.

Motivated by $M = 1 * \Phi$, we apply Möbius inversion to Nathanson's function once more and introduce the second Möbius transform $\Psi(n) = (\mu * \Phi)(n)$. Equivalently, $\Phi(n) = \sum \Psi(d)$, where d runs through the divisors of n . Since $\Phi = \mu * M$, we also have $\Psi = (\mu * \mu) * M$. Thus M , Φ , and Ψ form a natural convolutional hierarchy, with each successive step obtained by a Möbius transform.

The first objective of this paper is to investigate the arithmetic structure of Ψ . We determine the prime-power values of $h = \mu * \mu$, derive an explicit divisor representation for $\Psi(n)$, and prove the fundamental positivity property $\Psi(n) > 0$ for every $n \geq 1$. We further obtain explicit formulas for primes, prime powers, semiprimes, and squarefree integers, together with lower bounds involving the prime divisors of n and an asymptotic estimate $\Psi(n) \sim 2^n$.

The second objective is to use this positivity to study the matrix $G_{\Phi(S)}$, whose (i, j) -entry is $\Phi(\gcd(x_i, x_j))$, which we call the Nathanson GCD matrix associated with S . General matrices of the form $[f(\gcd(x_i, x_j))]$ belong to the established theory of GCD and meet matrices (Beslin & Ligh, 1989; Korkee & Haukkanen, 2003; Oval, 2004). Accordingly, the general incidence-matrix machinery used here is classical; the arithmetic contribution specific to the present setting is the analysis and positivity of $\Psi = \mu * \Phi$, which makes that machinery effective for Nathanson's subset totient.

Using $\Phi(n) = \sum \Psi(d)$, where d runs through the divisors of n , we obtain a divisor-incidence factorization $G_{\Phi(S)} = E_S D \Psi E_S^T$. Since $\Psi(d) > 0$, this representation implies that $G_{\Phi(S)}$ is positive definite for every finite set of distinct positive integers. For factor-closed sets it yields the product determinant $\det G_{\Phi(S)} = \prod_{x \in S} \Psi(x)$ and an explicit inverse formula; for arbitrary finite sets, Cauchy–Binet gives an exact determinant expansion.

The principal contributions are therefore the introduction and arithmetic analysis of the second Möbius transform $\Psi = \mu * \Phi$; the proof of its strict positivity; explicit formulas and asymptotic estimates; and the resulting positive-definiteness, determinant, and inverse results for Nathanson GCD matrices. This provides a direct link among relatively prime subset enumeration, Mersenne-type arithmetic functions, and structured GCD matrices.

The remainder of the paper is organized as follows. Section 2 recalls basic identities for Nathanson's totient function. Sections 3–5 develop the second Möbius transform, its positivity, explicit formulas, and bounds. Sections 6–8 study the associated GCD matrices, determinants, and inverses. Section 9 contains worked examples, followed by the Mersenne connection, conclusions, recommendations for future research, declarations, and references.

Basic Identities for Nathanson's Totient Function

Proposition 2.1. For every positive integer n ,

$$\sum_{d|n} \Phi(d) = 2^n - 1.$$

Proof. Take any nonempty subset $A \subseteq \{1, \dots, n\}$ and define

$$g = \gcd(\gcd A, n).$$

Then $g \mid n$. Dividing every element of A by g produces a nonempty subset of $\{1, \dots, n/g\}$ whose common gcd is relatively prime to n/g . Conversely, multiplying such a subset by g recovers A . Thus the nonempty subsets are partitioned by the value of g , and the class corresponding to g contains $\Phi(n/g)$ subsets. Hence

$$2^n - 1 = \sum_{g|n} \Phi(n/g) = \sum_{d|n} \Phi(d).$$

This proves the result.

Corollary 2.2. By Möbius inversion,

$$\Phi(n) = \sum_{d|n} \mu(d)(2^{n/d} - 1).$$

For $n > 1$, since $\sum_{d|n} \mu(d) = 0$,

$$\Phi(n) = \sum_{d|n} \mu(d)2^{n/d}.$$

Proposition 2.3. If p is prime and $a \geq 1$, then

$$\Phi(p^a) = 2^{p^a} - 2^{p^{a-1}}.$$

Proof. Among the divisors of p^a , the Möbius function is nonzero only at 1 and p . Substitution into Corollary 2.2 gives the formula.

For distinct primes p, q ,

$$\Phi(pq) = 2^{pq} - 2^p - 2^q + 2.$$

3. The Second Möbius Transform

Define

$$\Psi(n) = (\mu * \Phi)(n) = \sum_{d|n} \mu(d)\Phi(n/d).$$

By Möbius inversion,

$$\Phi(n) = \sum_{d|n} \Psi(d).$$

Let $M(n) = 2^n - 1$. Since $\Phi = \mu * M$,

$$\Psi = (\mu * \mu) * M.$$

Put $h = \mu * \mu$.

Lemma 3.1. The function h is multiplicative and, for every prime p ,

$$h(p^a) = \begin{cases} 1, & a = 0, \\ -2, & a = 1, \\ 1, & a = 2, \\ 0, & a \geq 3. \end{cases}$$

Proof. Dirichlet convolution preserves multiplicativity. Moreover,

$$h(p^a) = \sum_{j=0}^a \mu(p^j)\mu(p^{a-j}),$$

and $\mu(p^j) = 0$ for $j \geq 2$. The stated values follow.

Proposition 3.2. For every $n \geq 1$,

$$\Psi(n) = \sum_{d|n} h(d) 2^{n/d} - \mu(n).$$

Proof. Since $\Psi = h * M$,

$$\Psi(n) = \sum_{d|n} h(d) (2^{n/d} - 1).$$

Also,

$$\sum_{d|n} h(d) = (h * \mathbf{1})(n) = (\mu * \mu * \mathbf{1})(n) = \mu(n).$$

Substitution gives the result.

4. Positivity of Ψ

Lemma 4.1. For every $n \geq 1$,

$$\Phi(n) \geq 2^{n-1}.$$

Proof. Every subset of $\{1, \dots, n\}$ containing 1 has gcd 1 and is counted by $\Phi(n)$. Exactly 2^{n-1} subsets contain 1.

Theorem 4.2. For every positive integer n ,

$$1 \leq \Psi(n) \leq \Phi(n).$$

In particular, $\Psi(n) > 0$.

Proof. We argue by induction. For $n = 1$, $\Psi(1) = \Phi(1) = 1$. Assume the assertion for every proper divisor $d < n$. From $\Phi(n) = \sum_{d|n} \Psi(d)$,

$$\Psi(n) = \Phi(n) - \sum_{\substack{d|n \\ d < n}} \Psi(d).$$

Using the induction hypothesis $\Psi(d) \leq \Phi(d)$,

$$\Psi(n) \geq \Phi(n) - \sum_{\substack{d|n \\ d < n}} \Phi(d).$$

By Proposition 2.1,

$$\Psi(n) \geq 2\Phi(n) - 2^n + 1.$$

Lemma 4.1 gives $2\Phi(n) \geq 2^n$, hence $\Psi(n) \geq 1$. Since every proper-divisor term $\Psi(d)$ is positive, the defining identity also gives $\Psi(n) \leq \Phi(n)$.

Corollary 4.3. For $n \geq 3$,

$$\Psi(n) \geq 2^{n-2} + 1.$$

Proof. Besides all subsets containing 1, every subset containing both 2 and 3 but not 1 has gcd 1. There are 2^{n-3} such subsets. Hence

$$\Phi(n) \geq 2^{n-1} + 2^{n-3}.$$

Substituting into $\Psi(n) \geq 2\Phi(n) - 2^n + 1$ gives the result.

5. Explicit Formulas and Prime-Divisor Bounds

Proposition 5.1. If p is prime, then

$$\Psi(p) = 2^p - 3.$$

Proof. $\Psi(p) = \Phi(p) - \Phi(1) = (2^p - 2) - 1$.

Proposition 5.2. If $a \geq 2$, then

$$\Psi(p^a) = 2^{p^a} - 2 \cdot 2^{p^{a-1}} + 2^{p^{a-2}}.$$

Proof. By Lemma 3.1, only $1, p, p^2$ contribute, and $\mu(p^a) = 0$ for $a \geq 2$.

Proposition 5.3. For distinct primes $p < q$,

$$\Psi(pq) = 2^{pq} - 2^{q+1} - 2^{p+1} + 7.$$

Proof. The relevant divisors are $1, p, q, pq$, with h -values $1, -2, -2, 4$, and $\mu(pq) = 1$. Thus

$$\Psi(pq) = 2^{pq} - 2 \cdot 2^q - 2 \cdot 2^p + 4 \cdot 2 - 1,$$

which simplifies to the stated formula.

Proposition 5.4. If n is squarefree, then

$$\Psi(n) = \sum_{d|n} (-2)^{\omega(d)} 2^{n/d} - \mu(n).$$

Proof. For square free d , multiplicativity and Lemma 3.1 give $h(d) = (-2)^{\omega(d)}$. Substitute into Proposition 3.2.

Theorem 5.5. For $n > 1$,

$$\Psi(n) \geq 2^n - 2 \sum_{\substack{p|n \\ p \text{ prime}}} 2^{n/p} + 2\omega(n) - 1.$$

Proof. A subset is excluded from $\Phi(n)$ only if all its elements are divisible by at least one prime $p \mid n$. For fixed p , there are $2^{n/p} - 1$ nonempty subsets consisting entirely of multiples of p . By the union bound,

$$(2^n - 1) - \Phi(n) \leq \sum_{\substack{p|n \\ p \text{ prime}}} (2^{n/p} - 1).$$

Therefore

$$\Phi(n) \geq 2^n - 1 + \omega(n) - \sum_{\substack{p|n \\ p \text{ prime}}} 2^{n/p}.$$

Combining this with $\Psi(n) \geq 2\Phi(n) - 2^n + 1$ proves the result.

Corollary 5.6. If $p = P^-(n)$ is the smallest prime divisor of n , then

$$\Psi(n) \geq 2^n - 2\omega(n)2^{n/p} + 2\omega(n) - 1.$$

Proof. For every prime $q \mid n$, $q \geq p$, hence $2^{n/q} \leq 2^{n/p}$. Apply Theorem 5.5.

Corollary 5.7. As $n \rightarrow \infty$,

$$\Psi(n) \sim 2^n.$$

Indeed,

$$\frac{\Psi(n)}{2^n} = 1 + O(\omega(n)2^{-n/2}).$$

6. Nathanson GCD Matrices

Let $S = \{x_1, \dots, x_r\}$ be a finite set of distinct positive integers and define

$$G_\Phi(S) = \left[\Phi(\gcd(x_i, x_j)) \right]_{i,j=1}^r.$$

Let $D(S)$ be the set of all positive divisors of elements of S . Define the divisor-incidence matrix E by

$$E_{i,d} = \begin{cases} 1, & d \mid x_i, \\ 0, & d \nmid x_i, \end{cases}$$

and let

$$D_\Psi = \text{diag}(\Psi(d) : d \in D(S)).$$

Theorem 6.1.

$$G_\Phi(S) = ED_\Psi E^T.$$

Proof. The (i, j) entry of the right-hand side is

$$\sum_{\substack{d \mid x_i \\ d \mid x_j}} \Psi(d) = \sum_{d \mid \gcd(x_i, x_j)} \Psi(d) = \Phi(\gcd(x_i, x_j)),$$

which is the (i, j) entry of $G_\Phi(S)$.

Lemma 6.2. The divisor-incidence matrix E has full row rank.

Proof. Order $x_1 < \dots < x_r$ and select the r columns indexed by $x_1, \dots, x_r$. The resulting square matrix has entry 1 when $x_j \mid x_i$. If $j > i$, then $x_j > x_i$, so $x_j \nmid x_i$. Hence this submatrix is lower triangular with diagonal entries 1, and its determinant is 1.

Theorem 6.3. For every finite set S of distinct positive integers, $G_\Phi(S)$ is positive definite.

Proof. Let $z \neq 0$. By Theorem 6.1,

$$z^T G_\Phi(S) z = (E^T z)^T D_\Psi (E^T z).$$

Lemma 6.2 implies $E^T z \neq 0$. By Theorem 4.2, every diagonal entry $\Psi(d)$ is positive, so D_Ψ is positive definite. Therefore, the displayed quadratic form is strictly positive

Consequently every eigenvalue and every principal minor of $G_\Phi(S)$ is positive, and

$$\det G_\Phi(S) > 0.$$

7. Determinant Formulas

Theorem 7.1. If S is factor closed, then

$$\det G_\phi(S) = \prod_{x \in S} \Psi(x).$$

Proof. For a factor-closed set, $D(S) = S$. After increasing ordering, E is lower triangular with diagonal entries 1, so $\det E = 1$. Taking determinants in $G_\phi(S) = ED_\psi E^T$ gives

$$\det G_\phi(S) = (\det E)^2 \det D_\psi = \prod_{x \in S} \Psi(x).$$

□

Corollary 7.2. For $S = \{1, 2, \dots, n\}$,

$$\det[\Phi(\gcd(i, j))]_{i, j=1}^n = \prod_{k=1}^n \Psi(k).$$

Theorem 7.3. For arbitrary S with $|S| = r$,

$$\det G_\phi(S) = \sum_{\substack{K \subseteq D(S) \\ |K|=r}} (\det E[:, K])^2 \prod_{d \in K} \Psi(d).$$

Proof. Write

$$G_\phi(S) = (ED_\psi^{1/2})(ED_\psi^{1/2})^T.$$

Applying the Cauchy-Binet formula to this Gram product gives the stated expansion.

8. Inverse on Factor-Closed Sets

Theorem 8.1. If S is factor closed, then

$$(G_\phi(S)^{-1})_{ij} = \sum_{\substack{x_k \in S \\ x_i | x_k, x_j | x_k}} \frac{\mu(x_k/x_i) \mu(x_k/x_j)}{\Psi(x_k)}.$$

Proof. Möbius inversion gives

$$(E^{-1})_{ij} = \begin{cases} \mu(x_i/x_j), & x_j | x_i, \\ 0, & x_j \nmid x_i. \end{cases}$$

Since

$$G_\phi(S)^{-1} = E^{-T} D_\psi^{-1} E^{-1},$$

taking the (i, j) entry gives the claimed formula.

9. Worked Examples

Example 9.1. Arithmetic Values of Ψ

$$\begin{aligned} \Psi(4) &= 2^4 - 2 \cdot 2^2 + 2 = 10, \\ \Psi(6) &= 2^6 - 2^4 - 2^3 + 7 = 47, \\ \Psi(8) &= 2^8 - 2^5 + 2^2 = 228, \end{aligned}$$

$$\begin{aligned}\Psi(9) &= 2^9 - 2^4 + 2 = 498, \\ \Psi(10) &= 2^{10} - 2^6 - 2^3 + 7 = 959.\end{aligned}$$

Example 9.2. A Factor-Closed Set

Let $S = \{1, 2, 3, 6\}$. Then

$$G_\Phi(S) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 1 & 1 & 6 & 6 \\ 1 & 2 & 6 & 54 \end{pmatrix}.$$

Moreover,

$$\Psi(1) = 1, \quad \Psi(2) = 1, \quad \Psi(3) = 5, \quad \Psi(6) = 47.$$

Therefore

$$\det G_\Phi(\{1, 2, 3, 6\}) = 1 \cdot 1 \cdot 5 \cdot 47 = 235.$$

Example 9.3. Another Factor-Closed Set

For $S = \{1, 2, 3, 4\}$,

$$\det G_\Phi(S) = \Psi(1)\Psi(2)\Psi(3)\Psi(4) = 1 \cdot 1 \cdot 5 \cdot 10 = 50.$$

Example 9.4. A Non-Factor-Closed Set

Take $S = \{2, 3, 6\}$. Then

$$G_\Phi(S) = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 6 & 6 \\ 2 & 6 & 54 \end{pmatrix},$$

and a direct calculation gives

$$\det G_\Phi(\{2, 3, 6\}) = 470 > 0.$$

Thus, factor closure is unnecessary for positive definiteness; it is needed only for the simple product determinant formula.

10. Mersenne Connection

Since $M(n) = 2^n - 1$ and $M = \mathbf{1} * \Phi$,

$$M(\gcd(x_i, x_j)) = \sum_{d|\gcd(x_i, x_j)} \Phi(d).$$

Hence

$$\left[M(\gcd(x_i, x_j)) \right] = E \operatorname{diag}(\Phi(d)) E^T.$$

For factor-closed S ,

$$\det \left[M \left(\gcd(x_i, x_j) \right) \right] = \prod_{x \in S} \Phi(x).$$

Thus, the arithmetic chain

$$M \rightarrow \Phi \rightarrow \Psi$$

links Mersenne GCD matrices with the Nathanson GCD matrices studied here.

Conclusion

The second Möbius transform $\Psi = \mu * \Phi$ is the arithmetic object controlling GCD matrices generated by Nathanson's totient function. Its positivity yields positive definiteness for arbitrary finite sets, while factor-closed sets admit exact determinant and inverse formulas. Natural further questions concern divisibility and p -adic properties of Ψ and the separate study of the LCM kernel $\Phi(\text{lcm}(x_i, x_j))$.

Recommendations and Future Research

The results obtained in this study suggest several directions for further investigation. In particular, the arithmetic properties of the second Möbius transform $\Psi = \mu * \Phi$ deserve deeper study, including its divisibility properties, congruence behavior, p -adic valuations, and possible characterizations of its extremal values. It would also be interesting to investigate spectral properties, eigenvalue bounds, and condition numbers of Nathanson GCD matrices.

A natural continuation is the study of the corresponding LCM matrices $[\Phi(\text{lcm}(x_i, x_j))]$, where questions concerning nonsingularity, inertia, and determinant structure are expected to differ substantially from the GCD case. Another direction is to examine analogous constructions arising from other subset-counting arithmetic functions and their successive Möbius transforms.

Finally, the connection $M \rightarrow \Phi \rightarrow \Psi$ between Mersenne numbers, Nathanson's totient function, and its second Möbius transform suggests that further arithmetic-matrix constructions based on this convolution hierarchy may provide new families of structured GCD and LCM matrices.

Scientific Ethics Declarations

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Conflict of Interest

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