

The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM), 2026

Volume 42, Pages 237-259

ICBASSET 2026: International Conference on Basic Sciences, Engineering and Technology

Tetra Neutrosophic Alpha Star Generalized $\# \Psi$ Mappings in Tetra Neutrosophic Topological Spaces

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Abstract: In 2026, R. Radha, R. Princy, and Arokia Pratheesha S. V. introduced Tetra Neutrosophic Sets and investigated their fundamental properties. In 2024, S. Pooja, M. Vignashwaren, and L. Vidyarani introduced the concept of $\alpha^* \mathcal{G}^\# \Psi$ -closed sets in topological spaces. In the same year, they further investigated $\alpha^* \mathcal{G}^\# \Psi$ -continuous and irresolute maps. Motivated by these developments, we introduce the concept of Tetra Neutrosophic $\alpha^* \mathcal{G}^\# \Psi$ -closed sets in Tetra Neutrosophic topological spaces and investigate their fundamental properties. Based on this new class of generalized closed sets, we introduce Tetra Neutrosophic $\alpha^* \mathcal{G}^\# \Psi$ -continuous, $\alpha^* \mathcal{G}^\# \Psi$ -irresolute, $\alpha^* \mathcal{G}^\# \Psi$ -open, and $\alpha^* \mathcal{G}^\# \Psi$ -closed mappings. Their basic properties and characterizations are established. Furthermore, Tetra Neutrosophic $\alpha^* \mathcal{G}^\# \Psi$ -homeomorphisms are introduced, and some of their fundamental properties are investigated. The obtained results extend the theory of generalized mappings to the Tetra Neutrosophic topological setting and provide a systematic framework for studying generalized transformations between TN -Top-Spaces.

Keywords: TN -Top-Space, $TN\alpha^* \mathcal{G}^\# \Psi$ -closed set, $TN\alpha^* \mathcal{G}^\# \Psi$ -continuous Mapping, $TN\alpha^* \mathcal{G}^\# \Psi$ -irresolute Mapping, $TN\alpha^* \mathcal{G}^\# \Psi$ -open Mapping, $TN\alpha^* \mathcal{G}^\# \Psi$ -homeomorphism

Introduction

Topology is one of the fundamental branches of mathematics that provides a general framework for studying continuity, convergence, connectedness, compactness, and various types of mappings. In recent years, generalized closed sets and generalized open sets have received considerable attention because they provide broader and more flexible approaches to classical topological concepts. These generalized classes have also led to the development of several types of generalized continuous, irresolute, open, closed, and homeomorphic mappings.

Neutrosophic Set Theory, introduced by Smarandache, provides an effective mathematical framework for dealing with indeterminate, incomplete, and inconsistent information. Since its introduction, neutrosophic sets and their extensions have been extensively investigated in different areas of mathematics and applied sciences. In particular, neutrosophic topological structures have been developed to extend classical topological concepts to environments involving uncertainty and indeterminacy.

Radha et al. (2026) introduced Tetra Neutrosophic Sets and investigated their fundamental properties. Tetra Neutrosophic Sets provide a generalized framework for representing information through multiple truth, indeterminacy, and falsity membership components. This development provides a suitable setting for extending various generalized topological concepts to a richer neutrosophic environment.

Pooja et al. (2024) introduced the concept of $\alpha^* \mathcal{G}^\# \Psi$ -closed sets in topological spaces and investigated their fundamental properties. In the same year, they further studied $\alpha^* \mathcal{G}^\# \Psi$ -continuous and irresolute maps associated

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- Selection and peer-review under responsibility of the Organizing Committee of the Conference

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with these generalized closed sets. These developments motivated us to investigate the corresponding concepts in the framework of TN -Top-Spaces.

Motivated by the above developments, in this paper we introduce and study the concept of $TN\alpha^*g^\# \psi$ -closed sets in TN -Top-Spaces. We investigate their fundamental properties and establish several relationships with other generalized closed sets. We also introduce the corresponding notion of $TN\alpha^*g^\# \psi$ -open sets and study their basic properties.

Furthermore, based on the newly introduced class of $TN\alpha^*g^\# \psi$ -closed sets, we introduce $TN\alpha^*g^\# \psi$ -continuous mappings and $TN\alpha^*g^\# \psi$ -irresolute mappings. We investigate their fundamental properties, characterize these mappings under suitable conditions, and establish relationships between the different classes of generalized mappings. In addition, we introduce $TN\alpha^*g^\# \psi$ -open mappings and $TN\alpha^*g^\# \psi$ -closed mappings and study their basic properties.

Finally, we introduce the concept of a $TN\alpha^*g^\# \psi$ -homeomorphism and investigate some of its fundamental properties. The results obtained in this paper extend the existing theory of generalized closed sets and generalized mappings to the Tetra Neutrosophic topological setting. Thus, the present work provides a systematic mathematical framework for studying generalized transformations between Tetra Neutrosophic topological spaces and contributes to the continuing development of generalized neutrosophic topology.

2. Tetra Neutrosophic –Closed and Open Sets

Definition 2.1. Let R be a universe. A Neutrosophic set M on R can be defined as follows:

$$M = \{ \langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R \} \text{ where } T_M, I_M, F_M : U \rightarrow [0, 1] \text{ and } 0 \leq T_M(r) + I_M(r) + F_M(r) \leq 3.$$

Here, $T_M(r)$ is the degree of membership, $I_M(r)$ is the degree of indeterminacy and $F_M(r)$ is the degree of non-membership. Here, $T_M(r)$ and $F_M(r)$ are dependent neutrosophic components and $I_M(r)$ is an independent component.

Definition 2.2. Let R be a universe. A Neutrosophic Pythagorean set (briefly NP-set) M with T and F as dependent Neutrosophic Pythagorean components and U as an independent component for M on R is an object of the form $M = \{ \langle r, T_M(r), U_M(r), F_M(r) \rangle : r \in R \}$ where $T_M(r) + F_M(r) \leq 1$, $(T_M(r))^2 + (F_M(r))^2 \leq 1$, and $0 \leq (T_M(r))^2 + (U_M(r))^2 + (F_M(r))^2 \leq 2$, for all $r \in R$. We note that $T_M(r)$ is the truth membership, $U_M(r)$ is indeterminacy membership, and $F_M(r)$ is the false membership.

Definition 2.3. Let R be a non-empty set (universe). A Tetra Neutrosophic set (briefly TN-set) M on R is an object of the form: $M = \{ \langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R \}$ where $T_M(r), I_M(r), F_M(r) \in [0, 1]$, $0 \leq (T_M(r))^4 + (I_M(r))^4 + (F_M(r))^4 \leq 1$, for all $r \in R$. $T_M(r)$ is the truth membership, $I_M(r)$ is an indeterminacy membership and $F_M(r)$ is the false membership.

$TN(R)$ presents the family of all TN -sets on R .

Definition 2.4. The complement of a TN -set $M = \{ \langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R \}$ is defined and denoted by $M^c = \{ \langle r, F_M(r), 1 - I_M(r), T_M(r) \rangle : r \in R \}$

Definition 2.5. Let $M = \{\langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R\}$ and $Q = \{\langle r, T_Q(r), I_Q(r), F_Q(r) \rangle : r \in R\}$ be two TN-sets on the universe R . Then the union, the intersection and the difference of two TN-sets can be defined by $M \cup Q = \{\langle r, \max(T_M(r), T_Q(r)), \min(I_M(r), I_Q(r)), \min(F_M(r), F_Q(r)) \rangle : r \in R\}$
 $M \cap Q = \{\langle r, \min(T_M(r), T_Q(r)), \max(I_M(r), I_Q(r)), \max(F_M(r), F_Q(r)) \rangle : r \in R\}$
 $M - Q = \{\langle r, \min(T_M(r), F_Q(r)), \min(I_M(r), 1 - I_Q(r)), \max(F_M(r), T_Q(r)) \rangle : r \in R\}$

Definition 2.6. Let $M = \{\langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R\}$ and $Q = \{\langle r, T_Q(r), I_Q(r), F_Q(r) \rangle : r \in R\}$ be two TN-sets on the universe R . Then M is contained in Q (i.e) $M \subseteq Q$ if $T_M \leq T_Q$, $I_M \geq I_Q$ and $F_M \geq F_Q$.

Definition 2.7. Let R be a non-empty set. The TN-empty set on R , denoted by 0_{TN} , is defined by $0_{TN} = \{\langle r, 0, 1, 1 \rangle : r \in R\}$. The TN-universal set on R , denoted by 1_{TN} , is defined by $1_{TN} = \{\langle r, 1, 0, 0 \rangle : r \in R\}$.

Theorem 2.8. Let M , P and Q be any TN-sets on the universe R . Then

- (1) $M \cap 0_{TN} = 0_{TN}$ $0_{TN} = \{\langle 0, 1, 1 \rangle : r \in R\}$ (2) $M \cup 1_{TN} = 1_{TN}$ $1_{TN} = \{\langle 1, 0, 0 \rangle : r \in R\}$
- (3) $M \cup P = P \cup M$ (4) $M \cap P = P \cap M$ (5) $(M \cup P) \cup Q = M \cup (M \cup Q)$
- (6) $(M \cap P) \cap Q = M \cap (M \cap Q)$ (7) $(M^c)^c = M$ (8) $M \cup (M \cap Q) = M$ (9) $M \cap (M \cup Q) = M$
- (10) $M \cup (P \cap Q) = (M \cup P) \cap (M \cup Q)$ (11) $M \cap (P \cup Q) = (M \cap P) \cup (M \cap Q)$
- (12) $(M \cup Q)^c = M^c \cap Q^c$ (13) $(M \cap Q)^c = M^c \cup Q^c$ (14) $(0_{TN})^c = 1_{TN}$ (15) $(1_{TN})^c = 0_{TN}$ (16)
- $M \cup 0_{TN} = M$ (17) $M \cup 1_{TN} = 1_{TN}$ (18) $M \cap 0_{TN} = 0_{TN}$ (19) $M \cap 1_{TN} = M$ (20) $M \subseteq Q \Leftrightarrow M \cap Q = M$
- (21) $M \subseteq Q \Leftrightarrow M \cup Q = Q$.

Definition 2.10. A Tetra Neutrosophic topology (TN-Topology) on a nonempty set R is a $T_{TN} \subseteq TN(R)$ is a family T_{TN} of TN-sets on R satisfying the following axioms

- (1) $0_{TN}, 1_{TN} \in T_{TN}$
- (2) $K \cap L \in T_{TN}$ for any $K, L \in T_{TN}$
- (3) $\bigcup_{\lambda \in \Lambda} M_\lambda \in T_{TN}$ for any $\{M_\lambda : \lambda \in \Lambda\} \subseteq T_{TN}$

The pair (R, T_{TN}) is called a TN-Top-Space.

Definition 2.11. Let (R, T_{TN}) be a TN-Top-Space. and $M \in T_{TN}$. Then M is called a TN-open set in R . The complement of a TN-open set M is called a TN-closed set in R and it is denoted by M^c .

Definition 2.12. Let (R, T_{TN}) be a TN-Top-Space and $M = \{\langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R\}$ be a TN-set in R . Then the TN-interior and the TN-closure of M are denoted by $TN-Int(M)$ and $TN-CI(M)$ respectively and are defined as follows.

$$TN-Int(M) = \bigcap \{K : K \text{ is a TN-open set and } M \subseteq K\} \text{ and}$$

$$TN-CI(M) = \bigcup \{K : K \text{ is a TN-closed set and } K \subseteq M\}$$

Theorem 2.13. Let (R, T_{TN}) be a TN-Top-Space and let M and K be TN-set subset of R . Then the following statements hold.

- (i) $TN-Int(M)$ is a TN-open set in R .
- (ii) $TN-Int(0_{TN}) = 0_{TN}$ and $TN-Int(1_{TN}) = 1_{TN}$.
- (iii) $TN-CI(M)$ is a TN-closed set in R .

- (iv) $TN-Cl(0_{TN}) = 0_{TN}$ and $TN-Cl(1_{TN}) = 1_{TN}$.
- (v) M is TN -open set if and only if $M = TN-Int(M)$
- (vi) M is TN -closed set if and only if $M = TN-Cl(M)$
- (vii) $TN-Int[TN-Int(M)] = TN-Int(M)$
- (viii) $TN-Cl[TN-Cl(M)] = TN-Cl(M)$
- (ix) $M \subseteq K$ implies that $TN-Int(M) \subseteq TN-Int(K)$
- (x) $M \subseteq K$ implies that $TN-Cl(M) \subseteq TN-Cl(K)$
- (xi) $TN-Int(M \cap K) = [TN-Int(M)] \cap [TN-Int(K)]$
- (xii) $TN-Cl(M \cap K) \subseteq [TN-Cl(M)] \cap [TN-Cl(K)]$
- (xiii) $[TN-Int(M)] \cup [TN-Int(K)] \subseteq TN-Int(M \cup K)$

Definition 2.14. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set. M is said to be a Tetra Neutrosophic Pre-open (briefly TNP -open) set if $M \subseteq TN-Int(TN-Cl(M))$. The family of all TNP -open sets in (R, T_{TN}) is denoted by $TNP-O(R, T_{TN})$. The complement of a TNP -open set is called a Tetra Neutrosophic Pre-closed (briefly TNP -closed) set in R . M is said to be a Fermatean Neutrosophic Pre-closed (briefly TNP -closed) set if $TN-Cl(TN-Int(M)) \subseteq M$.

Definition 2.15. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set. M is said to be a Tetra Neutrosophic Semi-open (briefly TNS -open) set if $M \subseteq TN-Cl(TN-Int(M))$. The family of all TNS -open sets in (R, T_{TN}) is denoted by $TNS-O(R, T_{TN})$. The complement of a TNS -open set is called a Tetra Neutrosophic Semi-closed (briefly TNS -closed) set in R . M is said to be a Tetra Neutrosophic Semi-closed (briefly TNS -closed) set if $TN-Int(TN-Cl(M)) \subseteq M$.

Definition 2.16. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set. M is said to be a Tetra Neutrosophic α -open (briefly $TN\alpha$ -open) set if $M \subseteq TN-Int(TN-Cl(TN-Int(M)))$. The family of all $TN\alpha$ -open sets in (R, T_{TN}) is denoted by $TN\alpha-O(R, T_{TN})$. The complement of a $TN\alpha$ -open set is called a Tetra Neutrosophic α -closed (briefly $TN\alpha$ -closed) set in R . M is said to be a Tetra Neutrosophic α -closed (briefly $TN\alpha$ -closed) set if $TN-Cl(TN-Int(TN-Cl(M))) \subseteq M$.

Proposition 2.17. A TN -open subset M of a TN -Top-Space (R, T_{TN}) is a $TN\alpha$ -open set in R if and only if M is a TNS -open set and TNP -open set.

Definition 2.18. Let (R, T_{TN}) be a TN -Top-Space and $M = \{ \langle r, T_M(r), I_M(r), F_M(r) \rangle : r \in R \}$ be a TN -set in R . Then the $TN\alpha$ -interior and the $TN\alpha$ -closure of M are denoted by $TN\alpha-Int(M)$ and $TN\alpha-Cl(M)$, and are defined as follows.

$$TN\alpha-Int(M) = \bigcup \{ K : K \text{ is a } TN\alpha\text{-open set and } K \subseteq M \} \quad \text{and}$$

$$TN\alpha-Cl(M) = \bigcap \{ L : L \text{ is a } TN\alpha\text{-closed set and } M \subseteq L \}$$

Theorem 2.19. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set in X . Then M is a $TN\alpha$ -open set in R if and only if there exists a TN -open set K in R such that $K \subseteq M \subseteq TN-Int(TN-Cl(K))$.

Proof. Let M be a $TN\alpha$ -open set in R . Then $M \subseteq TN-Int[TN-Cl(TN-Int(M))]$. Then this implies that $TN-Int(M) \subseteq M \subseteq TN-Int[TN-Cl(TN-Int(M))]$. Let $K = TN-Int(M)$. Then there exists a TN -open set K such that $K \subseteq M \subseteq TN-Int[TN-Cl(K)]$.

Conversely, suppose that there is a TN -open set K such that $K \subseteq M \subseteq TN-Int[TN-Cl(K)]$. Then $K = TN-Int(K) \subseteq TN-Int(M)$. Therefore $TN-Cl(K) \subseteq TN-Cl(TN-Int(M))$, then we obtain $TN-Int(TN-Cl(K)) \subseteq TN-Int[TN-Cl(TN-Int(M))]$. But by hypothesis, we have $K \subseteq M \subseteq TN-Int[TN-Cl(K)]$. Then $M \subseteq TN-Int[TN-Cl(TN-Int(M))]$. Therefore, M is a $TN\alpha$ -open set in R .

Theorem 2.20. The union of any family of $TN\alpha$ -open sets in a TN -Top-Space (R, T_{TN}) is a $TN\alpha$ -open set in R .

Proof. Let $\{M_j : j \in J\}$ be a family of $TN\alpha$ -open sets in R . Then $M_j \subseteq TN-Int(TN-Cl(TN-Int(M_j)))$, for each $j \in J$. Since $\bigcup_{j \in J} TN-Int(M_j) \subseteq TN-Int(\bigcup_{j \in J} M_j)$ and $\bigcup_{i \in I} TN-Cl(M_i) \subseteq TN-Cl(\bigcup_{i \in I} M_i)$. We have $\bigcup_{j \in J} M_j \subseteq \bigcup_{j \in J} TN-Int(TN-Cl(TN-Int(M_j))) \subseteq TN-Int(\bigcup_{j \in J} TN-Cl(TN-Int(M_j))) \subseteq TN-Int(TN-Cl(\bigcup_{j \in J} TN-Int(M_j))) \subseteq TN-Int(TN-Cl(TN-Int(\bigcup_{j \in J} M_j)))$. Hence, we conclude that $\bigcup_{j \in J} M_j$ is a $TN\alpha$ -open set in R .

Theorem 2.21. Let (R, T_{TN}) be a TN -Top-Space. Then the following statements hold.

- (i) Every TN -open (resp. TN -closed) set is a $TN\alpha$ -open (resp. $TN\alpha$ -closed) set.
- (ii) Every $TN\alpha$ -open (resp. $TNP\alpha$ -closed) set is a TNS -open (resp. TNS -closed) set.
- (iii) Every $TN\alpha$ -open (resp. $TN\alpha$ -closed) set is a TNP -open (resp. TNP -closed) set.

Lemma 2.22. Let (R, T_{TN}) be a TN -Top-Space and let M be a TN -set in R .

- (i) If M is a TN -open set, then $TNS-Cl(M) = TN-Int(TN-Cl(M))$.
- (ii) $TNS-Int(TN-Cl(M)) = TN-Cl(TN-Int(TN-Cl(M)))$.

Definition 2.23. Let (R, T_{TN}) be a TN -Top-Space and let M be a TN -set in R . Then M is called a

- (i) Tetra Neutrosophic Regular-closed (briefly TN -Regular-closed) if $M = TN-Cl(TN-Int(M))$.
- (ii) Tetra Neutrosophic Regular-open (briefly TN -Regular-open) if $M = TN-Int(TN-Cl(M))$.
- (iii) Tetra Neutrosophic β -closed (briefly $TN\beta$ -closed) if $TN-Int(TN-Cl(TN-Int(M))) \subseteq M$.
- (iv) Tetra Neutrosophic β -open (briefly $TN\beta$ -open) if $M \subseteq TN-Cl(TN-Int(TN-Cl(M)))$.
- (v) Tetra Neutrosophic $\mathbf{b}$ -closed (briefly TNb -closed) if $[TN-Cl(TN-Int(M))] \cap [TN-Int(TN-Cl(M))] \subseteq M$.
- (vi) Tetra Neutrosophic $\mathbf{b}$ -open (briefly TNb -open) if $M \subseteq [TN-Int(TN-Cl(M))] \cup [TN-Cl(TN-Int(M))]$.
- (vii) Tetra Neutrosophic π -closed (briefly $TN\pi$ -closed) set if M is the union of TNR -closed sets.
- (viii) Tetra Neutrosophic π -open (briefly $TN\pi$ -open) if M is the union of TNR -open sets.

- (ix) Tetra Neutrosophic Generalized closed (briefly TNg -closed) set if $TN-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (x) Tetra Neutrosophic Generalized semi-closed (briefly TNg_s -closed) set if $TNS-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (xi) Tetra Neutrosophic α Generalized closed (briefly $TN\alpha_g$ -closed) set if $TN\alpha-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (xii) Tetra Neutrosophic generalized pre-closed set (briefly TNg_p -closed) if $TNP-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (xiii) Tetra Neutrosophic generalized α closed set (briefly TNg_α -closed) if $TN\alpha-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is $TN\alpha$ -open set in R .
- (xiv) Tetra Neutrosophic $\ast$ generalized α closed set (briefly $TN^\ast g_\alpha$ -closed) if $TN-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TNg_α -open set in R .
- (xv) Tetra Neutrosophic semi generalized closed set (briefly TNs_g -closed) if $TNS-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TNS -open set in R .
- (xvi) Tetra Neutrosophic generalized semi pre closed set (briefly TNg_{sp} -closed) if $TNSP-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (xvii) Tetra Neutrosophic generalized $\ast$ semi pre closed set (briefly $TNg^\ast_{sp}$ -closed) if $TNSP-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TNg -open set in R .
- (xviii) Tetra Neutrosophic generalized pre regular closed set (briefly TNg_{pr} -closed) if $TNP-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -regular open set in R .
- (xix) Tetra Neutrosophic ψ closed (briefly $TN\psi$ -closed) set if $TNS-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TNs_g -open set in R .
- (xx) Tetra Neutrosophic ψ generalized closed (briefly $TN\psi_g$ -closed) set if $TN\psi-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is TN -open set in R .
- (xxi) Tetra Neutrosophic $g^\#\psi$ -closed closed (briefly $TNg^\#\psi$ -closed) set if $TN\psi-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is $TN\psi$ -open set in R .
- (xxii) Tetra Neutrosophic $TNg^\#\psi$ -closed set (briefly $TNg^\#\psi$ -closed) set if $TN\psi-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is $TN\psi$ -open set in R .
- (xxiii) Tetra Neutrosophic $TN\alpha g^\#\psi$ -closed set (briefly $TN\alpha g^\#\psi$ -closed) set if $TN\alpha-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is $TNg^\#\psi$ -open set in R .

3. Tetra Neutrosophic $\alpha^\ast g^\#\psi$ -Closed and Open Sets

We introduce $TN\alpha^\ast g^\#\psi$ -closed and $TN\alpha^\ast g^\#\psi$ -open sets and explore some of their properties.

Definition 3.1. A TN -set M in a TN -Top-Space (R, T_{TN}) is said to be a Tetra Neutrosophic $\alpha^\ast g^\#\psi$ -closed (briefly $TN\alpha^\ast g^\#\psi$ -closed) set if $TN\alpha-Cl(M) \subseteq H$ whenever $M \subseteq H$ and H is $TN\alpha g^\#\psi$ -open set in (R, T_{TN}) . The family of all $TN\alpha^\ast g^\#\psi$ -closed in (R, T_{FQN}) is denoted by $TN\alpha^\ast g^\#\psi-C(R, T_{TN})$.

Definition 3.2. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set in (R, T_{TN}) . Then M is said to be a $TN\alpha^\ast g^\#\psi$ -open set in (R, T_{TN}) if its complement $M^c = 1_{TN} - M$ is a $TN\alpha^\ast g^\#\psi$ -closed set in (R, T_{TN}) . The family of all $TN\alpha^\ast g^\#\psi$ -open sets in (R, T_{TN}) is denoted by $TN\alpha^\ast g^\#\psi-O(R, T_{TN})$.

Definition 3.3. Let (R, T_{TN}) be a TN -Top-Space. and M be a TN -set in (R, T_{TN}) . Then the Tetra Neutrosophic $\alpha^*g^\# \psi$ -interior of M (briefly, $TN\alpha^*g^\# \psi - \text{Int}(M)$) is defined as the union of all $TN\alpha^*g^\# \psi$ -open sets contained in M i.e., $TN\alpha^*g^\# \psi - \text{Int}(M) = \bigcup \{G : G \text{ is a } TN\alpha^*g^\# \psi \text{-open set in } (R, T_{TN}) \text{ \& } G \subseteq M\}$.

Theorem 3.4. Let (R, T_{TN}) be a TN -Top-Space. If M and K are TN -sets in R . Then the following results hold:

- (i) $TN\alpha^*g^\# \psi - \text{Int}(0_{TN}) = 0_{TN}$.
- (ii) $TN\alpha^*g^\# \psi - \text{Int}(1_{TN}) = 1_{TN}$.
- (iii) $TN\alpha^*g^\# \psi - \text{Int}(M) \subseteq M$.
- (iv) $M \subseteq K \Rightarrow TN\alpha^*g^\# \psi - \text{Int}(M) \subseteq TN\alpha^*g^\# \psi - \text{Int}(K)$.
- (v) $TN\alpha^*g^\# \psi - \text{Int}(TN\alpha^*g^\# \psi - \text{Int}(M)) = TN\alpha^*g^\# \psi - \text{Int}(M)$.
- (vi) $TN - \text{Int}(M) \subseteq TN\alpha^*g^\# \psi - \text{Int}(M)$.
- (vii) $[TN\alpha^*g^\# \psi - \text{Int}(M)] \cup [TN\alpha^*g^\# \psi - \text{Int}(K)] \subseteq TN\alpha^*g^\# \psi - \text{Int}(M \cup K)$.
- (viii) $TN\alpha^*g^\# \psi - \text{Int}(M \cap K) \subseteq [TN\alpha^*g^\# \psi - \text{Int}(M)] \cap [TN\alpha^*g^\# \psi - \text{Int}(K)]$.

Definition 3.5. Let (R, T_{TN}) be a TN -Top-Space and M be a TN -set in (R, T_{TN}) . Then the Tetra Neutrosophic $\alpha^*g^\# \psi$ -closure of M (briefly, $TN\alpha^*g^\# \psi - \text{Cl}(M)$) is defined as the intersection of all $TN\alpha^*g^\# \psi$ -closed sets in (R, T_{TN}) containing M i.e., $TN\alpha^*g^\# \psi - \text{Cl}(M) = \bigcap \{H : H \text{ is a } TN\alpha^*g^\# \psi \text{-closed set in } (R, T_{TN}) \text{ \& } M \subseteq H\}$.

Theorem 3.6. Let (R, T_{TN}) be a TN -Top-Space. Let M and K be TN -sets in R . Then the following results hold:

- (i) $TN\alpha^*g^\# \psi - \text{Cl}(0_{TN}) = 0_{TN}$.
- (ii) $TN\alpha^*g^\# \psi - \text{Cl}(1_{TN}) = 1_{TN}$.
- (iii) $M \subseteq TN\alpha^*g^\# \psi - \text{Cl}(M)$.
- (iv) $M \subseteq K \Rightarrow TN\alpha^*g^\# \psi - \text{Cl}(M) \subseteq TN\alpha^*g^\# \psi - \text{Cl}(K)$.
- (v) $TN\alpha^*g^\# \psi - \text{Cl}(TN\alpha^*g^\# \psi - \text{Cl}(M)) = TN\alpha^*g^\# \psi - \text{Cl}(M)$.
- (vi) $M \subseteq TN\alpha^*g^\# \psi - \text{Cl}(M) \subseteq TN - \text{Cl}(M)$.
- (vii) $[TN\alpha^*g^\# \psi - \text{Cl}(M)] \cup [TN\alpha^*g^\# \psi - \text{Cl}(K)] \subseteq TN\alpha^*g^\# \psi - \text{Cl}(M \cup K)$.
- (viii) $TN\alpha^*g^\# \psi - \text{Cl}(M \cap K) \subseteq [TN\alpha^*g^\# \psi - \text{Cl}(M)] \cap [TN\alpha^*g^\# \psi - \text{Cl}(K)]$.

Theorem 3.7. Every TN -closed (resp. TN -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Consider a TN -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a $TN\alpha^*g^\# \psi$ -open set in (R, T_{TN}) . Since M is a TN -closed set in (R, T_{TN}) , $TN - \text{Cl}(M) = M$. But $TN\alpha^*g^\# \psi - \text{Cl}(M) \subseteq TN - \text{Cl}(M) \Rightarrow TN\alpha^*g^\# \psi - \text{Cl}(M) \subseteq M$. Therefore, M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) .

Theorem 3.8. Every $TN\alpha$ -closed (resp. $TN\alpha$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a $TN\alpha g^\# \psi$ -open set in (R, T_{TN}) . Since M is a $TN\alpha$ -closed set in (R, T_{TN}) , $TN\alpha -Cl(M) = M$. Therefore $TN\alpha -Cl(M) \subseteq H$. Hence, M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) .

Theorem 3.9. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TNgs$ -closed (resp. $TNgs$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -open set in (R, T_{TN}) . As every TN -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. But $TNS -Cl(M) \subseteq TN\alpha -Cl(M)$, then $TNS -Cl(M) \subseteq H$. Therefore, M is a $TNgs$ -closed set in (R, T_{TN}) .

Theorem 3.10. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TNsg$ -closed (resp. $TNsg$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -open set in (R, T_{TN}) . As every TN -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Also, $TNS -Cl(M) \subseteq TN\alpha -Cl(M)$, then $TNS -Cl(M) \subseteq H$. Therefore, M is a $TNsg$ -closed set in (R, T_{TN}) .

Theorem 3.11. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\alpha g$ -closed (resp. $TN\alpha g$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -open set in (R, T_{TN}) . As every TN -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Therefore, M is a $TN\alpha g$ -closed set in (R, T_{TN}) .

Theorem 3.12. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TNgsp$ -closed (resp. $TNgsp$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -open set in (R, T_{TN}) . As every TN -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Also, $TNSP -Cl(M) \subseteq TN\alpha -Cl(M)$. This implies $TNSP -Cl(M) \subseteq H$. Therefore, M is a $TNgsp$ -closed set in (R, T_{TN}) .

Theorem 3.13. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a TNg^*sp -closed (resp. TNg^*sp -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TNg -open set in (R, T_{TN}) . As every TNg -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M

is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Also, $TNSP -Cl(M) \subseteq TN\alpha -Cl(M)$. This implies $TNSP -Cl(M) \subseteq H$. Therefore, M is a TNg^*sp -closed set in (R, T_{TN}) .

Theorem 3.14. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TNgpr$ -closed (resp. $TNgpr$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -regular open set in (R, T_{TN}) . As every TN -regular open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Also, $TNP -Cl(M) \subseteq TN\alpha -Cl(M)$. This implies $TNP -Cl(M) \subseteq H$. Therefore, M is a $TNgpr$ -closed set in (R, T_{TN}) .

Theorem 3.15. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\psi g$ -closed (resp. $TN\psi g$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a TN -open set in (R, T_{TN}) . As every TN -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Here M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) , so $TN\alpha -Cl(M) \subseteq H$. Also, $TN\psi -Cl(M) \subseteq TN\alpha -Cl(M)$. This implies $TN\psi -Cl(M) \subseteq H$. Therefore, M is a $TN\psi g$ -closed set in (R, T_{TN}) .

Theorem 3.16. Every $TNg^\#$ -closed (resp. $TNg^\#$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Consider a $TNg^\#$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a $TN\alpha g^\# \psi$ -open set in (R, T_{TN}) . As M is $TNg^\#$ -closed set, then $TN -Cl(M) \subseteq H$. But $TN\alpha -Cl(M) \subseteq TN -Cl(M)$. This implies $TN\alpha -Cl(M) \subseteq H$. Therefore, M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) .

Theorem 3.17. Every $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TNg^\# \psi$ -closed (resp. $TNg^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha^*g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a $TN\psi$ -open set in (R, T_{TN}) . As every $TN\psi$ -open set is $TN\alpha g^\# \psi$ -open set, H is $TN\alpha g^\# \psi$ -open set. Since M is $TN\alpha^*g^\# \psi$ -closed set, $TN\alpha -Cl(M) \subseteq H$. But $TN\psi -Cl(M) \subseteq TN\alpha -Cl(M)$. This implies $TN\alpha -Cl(M) \subseteq H$. Therefore, M is a $TNg^\# \psi$ -closed set in (R, T_{TN}) .

Theorem 3.18. Every $TN\alpha g^\# \psi$ -closed (resp. $TN\alpha g^\# \psi$ -open) set in a TN -Top-Space (R, T_{TN}) is a $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Consider a $TN\alpha g^\# \psi$ -closed set M in a TN -Top-Space (R, T_{TN}) . Assume that $M \subseteq H$ and H be a $TN\alpha g^\# \psi$ -open set in (R, T_{TN}) . As every $TN\alpha g^\# \psi$ -open set is $TNg^\# \psi$ -open set, H is $TNg^\# \psi$ -open set. Since M is $TN\alpha g^\# \psi$ -closed set, $TN\alpha -Cl(M) \subseteq H$. Therefore, M is a $TN\alpha^*g^\# \psi$ -closed set in (R, T_{TN}) .

Theorem 3.19. The intersection (resp. union) of two $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) sets in a TN -Top-Space (R, T_{TN}) is a $TN\alpha^*g^\# \psi$ -closed (resp. $TN\alpha^*g^\# \psi$ -open) set in (R, T_{TN}) .

Proof. Let M and K be $TN\alpha^*g^\# \psi$ -closed sets and $M \cup K \subseteq H$ where H is $TN\alpha^*g^\# \psi$ -open set. Then $M \subseteq H$ and $K \subseteq H$. As M and K are $TN\alpha^*g^\# \psi$ -closed sets, $TN\alpha -Cl(M) \subseteq H$ and $TN\alpha -Cl(K) \subseteq H$. Also $TN\alpha -Cl(M \cup K) = TN\alpha -Cl(M) \cup TN\alpha -Cl(K) \subseteq H \Rightarrow TN\alpha -Cl(M \cup K) \subseteq H$. Thus it follows that $M \cup K$ is $TN\alpha^*g^\# \psi$ -closed set. Hence, the union of two $TN\alpha^*g^\# \psi$ -closed sets is $TN\alpha^*g^\# \psi$ -closed set.

Theorem 3.20. Let M be a TN -open set and K be a $TN\alpha^*g^\# \psi$ -open set in a TN -Top-Space. (R, T_{TN}) . Then $M \cap K$ is $TN\alpha^*g^\# \psi$ -open set.

Proof. Let M be a TN -open set and K be a $TN\alpha^*g^\# \psi$ -open set in a TN -Top-Space (R, T_{TN}) . As every TN -open set is $TN\alpha^*g^\# \psi$ -open set, M is $TN\alpha^*g^\# \psi$ -open set. Since the intersection of two $TN\alpha^*g^\# \psi$ -open sets is $TN\alpha^*g^\# \psi$ -open set. Hence $M \cap K$ is $TN\alpha^*g^\# \psi$ -open set.

Example 3.21. Let $R = \{a, b, c\}$. Let G, H and M be TN -sets given by

$$G = \{\langle a, 0.7, 0.10, 0.10 \rangle, \langle b, 0, 0, 1 \rangle, \langle c, 0, 0, 1 \rangle\}, \quad H = \{\langle a, 1, 0, 0 \rangle, \langle b, 0.60, 0.20, 0.20 \rangle, \langle c, 0, 0, 1 \rangle\},$$

$$M = \{\langle a, 0, 0, 1 \rangle, \langle b, 0.60, 0.20, 0.20 \rangle, \langle c, 0, 0, 1 \rangle\}. \quad \text{Clearly} \quad (0.70)^4 + (0.10)^4 + (0.10)^4 = 0.2403 \leq 1, \text{ and}$$

$$(0.60)^4 + (0.20)^4 + (0.20)^4 = 0.1328 \leq 1. \text{ Thus, all the above triples satisfy the condition}$$

$$T^4 + I^4 + F^4 \leq 1. \text{ Consider the tetra neutrosophic Topology}$$

$$T_{TN} = \{0_{TN}, G, H, 1_{TN}\}. \text{ Since } M \text{ is different from the corresponding } TN\text{-closed sets:}$$

$$0_{TN}, G^c, H^c, 1_{TN}. \text{ Hence } M \text{ is not } TN\text{-closed. Now, } TN\text{-Int}(M) = 0_{TN}. \text{ Therefore,}$$

$$TN\text{-Cl}(TN\text{-Int}(M)) = TN\text{-Cl}(0_{TN}) = 0_{TN}, \text{ and hence}$$

$$TN\text{-In}(TN\text{-Cl}(TN\text{-Int}(M))) = 0_{TN}. \text{ Furthermore, the } TN\text{-complement of } M \text{ is } TN\alpha\text{-open. Hence } M \text{ is}$$

$$TN\alpha\text{-closed and consequently } TN\alpha\text{-Cl}(M) = M.$$

Let H be any $TN\alpha^*g^\# \psi$ -open set such that $M \subseteq H$. Since $TN\alpha -Cl(M) = M$. We immediately obtain

$$TN\alpha -Cl(M) = M \subseteq H. \text{ Therefore, by the definition of a } TN\alpha^*g^\# \psi\text{-closed set, } M \text{ is } TN\alpha^*g^\# \psi\text{-closed. Thus,}$$

by Theorem 3.18, M is $TN\alpha^*g^\# \psi$ -closed which is not TN -closed. Hence, $TN\alpha$ -closed $\Rightarrow$

$TN\alpha^*g^\# \psi$ -closed, but the converse implication does not hold.

4. Tetra Neutrosophic $\alpha^*g^\# \psi$ -Continuous Mappings

In this section, we introduce the concepts of Tetra Neutrosophic $\alpha^*g^\# \psi$ -continuous (briefly, $TN\alpha^*g^\# \psi$ -continuous) mappings in TN -Top-Spaces. Also, we study some of the main results depending on $TN\alpha^*g^\# \psi$ -open and $TN\alpha^*g^\# \psi$ -closed sets.

Definition 4.1. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a mapping. Then ξ is called a $TN\alpha^*g^\# \psi$ -continuous mapping if $\xi^{-1}(H)$ is a $TN\alpha^*g^\# \psi$ -open set in R for every TN -open set H in Z .

Theorem 4.2. Every TN -continuous mapping is a $TN\alpha^*g^\# \psi$ -continuous mapping.

Proof. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be TN -continuous mapping. Let H be a TN -open set in (Z, Γ_{TN}) . Then $\xi^{-1}(H)$ is TN -open set in (R, T_{TN}) . Since every TN -open set is $TN\alpha^*g^\# \psi$ -open, $\xi^{-1}(H)$ is $TN\alpha^*g^\# \psi$ -open set in (R, T_{TN}) . Hence ξ is a $TN\alpha^*g^\# \psi$ -continuous mapping.

Theorem 4.3. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha^*g^\# \psi$ -continuous map and $\mu: (Z, \Gamma_{TN}) \rightarrow (W, \Psi_{TN})$ be a TN -continuous mappings. Then prove that $\mu \circ \xi: (R, T_{TN}) \rightarrow (W, \Psi_{TN})$ is a $TN\alpha^*g^\# \psi$ -continuous mapping.

Proof. Let H be a TN -open set in W . Since $\mu: (Z, \Gamma_{TN}) \rightarrow (W, \Psi_{TN})$ is TN -continuous, $\mu^{-1}(H)$ is TN -open set in Z . Since ξ is a $TN\alpha^*g^\# \psi$ -continuous mapping, $\xi^{-1}[\mu^{-1}(H)]$ is a $TN\alpha^*g^\# \psi$ -open set in R . But $\xi^{-1}[\mu^{-1}(H)] = (\mu \circ \xi)^{-1}(H)$. Then $(\mu \circ \xi)^{-1}(H)$ is a $TN\alpha^*g^\# \psi$ -open set in R . Therefore, $\mu \circ \xi$ is a $TN\alpha^*g^\# \psi$ -continuous mapping.

Theorem 4.4. Prove that $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is a $TN\alpha^*g^\# \psi$ -continuous mapping if and only if $\xi^{-1}(H)$ is $TN\alpha^*g^\# \psi$ -closed set in R for every TN -closed set H in Z .

Proof. Let H be a TN -closed set in Z . Then H^C is TN -open set in Z . Since ξ is $TN\alpha^*g^\# \psi$ -continuous. Therefore $\xi^{-1}(H^C)$ is a $TN\alpha^*g^\# \psi$ -open set in R . Since $\xi^{-1}(H^C) = [\xi^{-1}(H)]^C$, $\xi^{-1}(H)$ is a $TN\alpha^*g^\# \psi$ -closed set in R .

Conversely, Let H be a TN -open set in Z . Then H^C is TN -closed set in Z . By assumption $\xi^{-1}(H^C)$ is $TN\alpha^*g^\# \psi$ -closed set in R . Since $\xi^{-1}(H^C) = [\xi^{-1}(H)]^C$, $\xi^{-1}(H)$ is $TN\alpha^*g^\# \psi$ -open set in R . Therefore, ξ is a $TN\alpha^*g^\# \psi$ -continuous mapping.

The following example shows that a $TN\alpha^*g^\# \psi$ -continuous function may not be a TN -continuous function.

Example 4.5. Let $R = \{p_1, p_2, p_3\}$, where $p_1 = (0.80, 0.20, 0.20)$, $p_2 = (0.60, 0.40, 0.20)$, $p_3 = (0.40, 0.30, 0.30)$. Let the Tetra Neutrosophic topology on R be $T_{TN} = \{0_{TN}, \{p_1\}, 1_{TN}\}$. Consider the set $A = \{p_1, p_2\}$. Then $TN-Int(A) = \{p_1\}$, $TN-Cl(TN-Int(A)) = 1_{TN}$, and $A \subseteq TN-Int(TN-Cl(TN-Int(A))) = 1_{TN}$. This shows that A is a $TN\alpha$ -open set. Since every $TN\alpha$ -open set is a $TN\alpha^*g^\# \psi$ -open set. Thus, A is a $TN\alpha^*g^\# \psi$ -open set. But $A \notin T_{TN}$, therefor, A is not TN -open. Let $Z = \{q_1, q_2\}$, where $q_1 = (0.70, 0.20, 0.20)$, $q_2 = (0.40, 0.40, 0.30)$, and let $\Gamma_{TN} = \{0_{TN}, \{q_1\}, 1_{TN}\}$ be the Tetra Neutrosophic topology on Z . Define the mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ by $\xi(p_1) = q_1$, $\xi(p_2) = q_1$, $\xi(p_3) = q_2$. Then $\xi^{-1}(\{q_1\}) = \{p_1, p_2\} = A$, and $\xi^{-1}(Z) = 1_{TN}$. Therefore, ξ is $TN\alpha^*g^\# \psi$ -continuous but not TN -continuous.

Theorem 4.6. A mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is a $TN\alpha^*g^\# \psi$ -continuous mapping if and only if $\xi(TN\alpha^*g^\# \psi-Cl(E)) \subseteq TN-Cl(\xi(E))$ for every TN -set E in R .

Proof. Let E be a TN -set in R and ξ be a $TN\alpha^*g^\# \psi$ -continuous mapping. We note that evidently $\xi(E) \subseteq TN\alpha^*g^\# \psi-Cl[\xi(E)]$. Then we obtain $E \subseteq \xi^{-1}[\xi(E)] \subseteq \xi^{-1}[TN\alpha^*g^\# \psi-Cl(\xi(E))]$ and $TN\alpha^*g^\# \psi-Cl(E) \subseteq TN\alpha^*g^\# \psi-Cl[\xi^{-1}(TN\alpha^*g^\# \psi-Cl(\xi(E)))]$. Since ξ is a $TN\alpha^*g^\# \psi$ -continuous mapping

and $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(E)]$ is a $\text{TN}\alpha^*g^\#\psi$ -closed set. Therefore, $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(\text{TN}\alpha^*g^\#\psi - \text{Cl}(\xi(E)))] = \xi^{-1}[\text{TN}\alpha^*g^\#\psi - \text{Cl}(\xi(E))]$. Hence, we have $\xi[\text{TN}\alpha^*g^\#\psi - \text{Cl}(E)] \subseteq \text{TN} - \text{Cl}[\xi(E)]$. Conversely, let $\xi[\text{TN}\alpha^*g^\#\psi - \text{Cl}(E)] \subseteq \text{TN} - \text{Cl}[\xi(E)]$, for each TN -set E in R . Let H be a TN -closed set in Z . Then $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(\xi^{-1}(H))] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Cl}(H) = H$. By assumption, by taking $E = \xi^{-1}(H)$, we obtain $\xi[\text{TN}\alpha^*g^\#\psi - \text{Cl}(\xi^{-1}(H))] \subseteq \text{TN} - \text{Cl}[\xi(\xi^{-1}(H))] \subseteq H$ and hence $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] \subseteq \xi^{-1}(H)$. Since $\xi^{-1}(H) \subseteq \text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)]$. Thus $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] = \xi^{-1}(H)$. This implies that $\xi^{-1}(H)$ is a $\text{TN}\alpha^*g^\#\psi$ -closed set in R . Therefore, by Theorem 4.4, ξ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping.

Theorem 4.7. Let $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ be a mapping. Then ξ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping if and only if $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] \subseteq \xi^{-1}[\text{TN} - \text{Cl}(H)]$ for each TN -set H in Z .

Proof. Suppose that $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping. Take H be any TN -set in Z . Clearly $H \subseteq \text{TN} - \text{Cl}(H)$ implies $\xi^{-1}(H) \subseteq \xi^{-1}[\text{TN}\alpha^*g^\#\psi - \text{Cl}(H)]$. Then $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(\text{TN} - \text{Cl}(H))]$. Since $\text{TN} - \text{Cl}(H)$ is TN -closed set in Z . So by Theorem 4.4, $\xi^{-1}[\text{TN} - \text{Cl}(H)]$ is a $\text{TN}\alpha^*g^\#\psi$ -closed set in R . Therefore, $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(\text{TN} - \text{Cl}(H))] = \xi^{-1}[\text{TN} - \text{Cl}(H)]$. Hence, we obtain $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] \subseteq \xi^{-1}[\text{TN} - \text{Cl}(H)]$. Conversely, let $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(H)] \subseteq \xi^{-1}[\text{TN} - \text{Cl}(H)]$ for every TN -set H in Z . Let K be a TN -closed set in Z . Since every TN -closed set is $\text{TN}\alpha^*g^\#\psi$ -closed set, $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi^{-1}(K)] \subseteq \xi^{-1}[\text{TN} - \text{Cl}(K)] = \xi^{-1}(K)$. This implies that $\xi^{-1}(K)$ is a $\text{TN}\alpha^*g^\#\psi$ -closed set in R . Thus, by Theorem 4.4, ξ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping.

Theorem 4.8. A mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping if and only if $\xi^{-1}[\text{TN} - \text{Int}(H)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(H)]$ for every TN -set H in Z .

Proof. Let H be any TN -set in Z and ξ be a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping. Then $\text{TN} - \text{Int}(H) \subseteq H$ implies $\xi^{-1}[\text{TN} - \text{Int}(H)] \subseteq \xi^{-1}(H)$. Therefore then this implies that $\text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(\text{TN} - \text{Int}(H))] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(H)]$. Since $\text{TN} - \text{Int}(H)$ is TN -open set in Z and ξ is $\text{TN}\alpha^*g^\#\psi$ -continuous Therefore, $\xi^{-1}[\text{TN} - \text{Int}(H)]$ is $\text{TN}\alpha^*g^\#\psi$ -open set in R . Hence, we get $\text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(\text{TN} - \text{Int}(H))] = \xi^{-1}[\text{TN} - \text{Int}(H)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(H)]$.

Conversely, let $\xi^{-1}[\text{TN} - \text{Int}(H)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(H)]$ for every TN -set H in Z . Let G be any TN -open set in Z and hence $\text{TN} - \text{Int}(G) = G$. By hypothesis, we have $\xi^{-1}[\text{TN} - \text{Int}(G)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(G)]$. Therefore. we obtain $\xi^{-1}(G) \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(G)]$. Also $\text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(G)] \subseteq [\xi^{-1}(G)]$. Consequently, $\text{TN}\alpha^*g^\#\psi - \text{Int}[\xi^{-1}(G)] = [\xi^{-1}(G)]$. This implies that $\xi^{-1}(G)$ is $\text{TN}\alpha^*g^\#\psi$ -open set in R . Hence we conclude that ξ is a $\text{TN}\alpha^*g^\#\psi$ -continuous mapping.

Definition 4.9. Let $p_{(t,i,f)}$ be a TN -point of a TN -Top-Space (R, T_{TN}) . A TN -set M of R is called TN -neighbourhood of $p_{(t,i,f)}$ if there exists a TN -open set G such that $p_{(t,i,f)} \in G \subseteq M$.

Theorem 4.10. Let $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ be a mapping. Then the following assertions are equivalent.

- (i) ξ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -continuous.
- (ii) For each TN -point $p_{(\tau, \lambda, \mathcal{E})}$ in R and every TN -neighbourhood M_p of $\xi(p_{(\tau, \lambda, \mathcal{E})})$, there exists a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set G_p in R such that $p_{(\tau, \lambda, \mathcal{E})} \in G_p \subseteq \xi^{-1}(M_p)$.
- (iii) For each TN -point $p_{(\tau, \lambda, \mathcal{E})}$ in R and every TN -neighbourhood H of $\xi(p_{(\tau, \lambda, \mathcal{E})})$, there exists a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set G in R such that $p_{(\tau, \lambda, \mathcal{E})} \in G$ and $\xi(G) \subseteq H$.

Proof. (i) $\Rightarrow$ (ii): Let $p_{(\tau, \lambda, \mathcal{E})}$ be a TN -point in R and let H be a TN -neighbourhood of TN -open set G in Z such that $\xi(p_{(\tau, \lambda, \mathcal{E})}) \in G \subseteq H$. Since ξ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -continuous, we know that $\xi^{-1}(G)$ is a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in R and $p_{(\tau, \lambda, \mathcal{E})} \in \xi^{-1}(G) \subseteq \xi^{-1}(H)$. This implies (ii) is true.

(ii) $\Rightarrow$ (iii): Let $p_{(\tau, \lambda, \mathcal{E})}$ be a TN -point in R and let H be a TN -neighbourhood of $\xi(p_{(\tau, \lambda, \mathcal{E})})$. The condition (ii) implies that there exists a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set G in R such that $p_{(\tau, \lambda, \mathcal{E})} \in G \subseteq \xi^{-1}(H)$. Thus $p_{(\tau, \lambda, \mathcal{E})} \in G$ and $\xi(G) \subseteq \xi[\xi^{-1}(H)] \subseteq H$. Hence (iii) is true.

(iii) $\Rightarrow$ (i): Let H be a TN -open set in Z and let $p_{(\tau, \lambda, \mathcal{E})} \in \xi^{-1}(H)$. Since H is TN -open set, $\xi(p_{(\tau, \lambda, \mathcal{E})}) \in H$, and so H is TN -neighbourhood of $\xi(p_{(\tau, \lambda, \mathcal{E})})$. It follows from (iii) that there exists a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set G in R such that $p_{(\tau, \lambda, \mathcal{E})} \in G$ and $\xi(G) \subseteq H$ so that $p_{(\tau, \lambda, \mathcal{E})} \in G \subseteq \xi^{-1}[\xi(G)] \subseteq \xi^{-1}(H)$. This implies by definition that $\xi^{-1}(H)$ is a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in R . Therefore, ξ is a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -continuous mapping.

Proof. Let $p_{(\tau, \lambda, \mathcal{E})}$ be a TN -point in R and let H be a TN -neighbourhood of $\xi(p_{(\tau, \lambda, \mathcal{E})})$. Then there exists a TN -open set E in Z such that $\xi(p_{(\tau, \lambda, \mathcal{E})}) \in E \subseteq H$. Since ξ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -continuous, we know that $G = \xi^{-1}(E)$ is a $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in R and $p_{(\tau, \lambda, \mathcal{E})} \in G = \xi^{-1}(E) \subseteq \xi^{-1}(H)$.

5. Tetra Neutrosophic $\alpha^*\mathcal{G}^\#\psi$ -Irresolute Mappings

In this section, we introduce the concept of Tetra Neutrosophic $\alpha^*\mathcal{G}^\#\psi$ -irresolute (briefly, $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -irresolute) mappings in TN -Top-Spaces. Also, we study some of the main results depending on $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open and $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -closed sets in TN -Top-Spaces.

Definition 5.1. Let a mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is called $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -irresolute if the inverse image of every $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in Z is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open in R .

Theorem 5.2. A mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -irresolute if and only if the inverse image of every $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -closed set in (Z, Γ_{TN}) is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -closed set in (R, T_{TN}) .

Proof. Let H be any $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -closed set in Z . Then H^C is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in Z . Since ξ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -irresolute, $\xi^{-1}(H^C)$ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -open set in R and $\xi^{-1}(H^C) = [\xi^{-1}(H)]^C$ which implies that $\xi^{-1}(H)$ is $\text{TN}\alpha^*\mathcal{G}^\#\psi$ -closed set in R .

Conversely, Let G be any $\text{TN}\alpha^*\text{g}^\#\psi$ -open set in Z . Then G^c is $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in Z . Thus $\xi^{-1}(G^c)$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in R and $\xi^{-1}(G^c) = [\xi^{-1}(G)]^c$ which implies that $\xi^{-1}(G)$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -open set in R . Hence $\xi: U \rightarrow W$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute.

Theorem 5.3. Every $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -continuous.

Proof. Let H be a TN -open set in Z . Since every TN -open set is $\text{TN}\alpha^*\text{g}^\#\psi$ -open, H is $\text{TN}\alpha^*\text{g}^\#\psi$ -open. Since ξ is $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute, $\xi^{-1}(H)$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -open in R . Therefore ξ is $\text{TN}\alpha^*\text{g}^\#\psi$ -continuous.

Theorem 5.4. A mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping if and only if $\xi(\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(M)) \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi(M))$ for every TN -set M in (R, T_{TN}) .

Proof. Let M be a TN -set in R and ξ be a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping. Then evidently $\xi(M) \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi(M)]$. Then we have $M \subseteq \xi^{-1}[\xi(M)] \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi(M))]$ and hence $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(M) \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi(M)))]$. Since ξ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping and $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi(M)]$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set. Therefore $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi(M)))] = \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi(M))]$. So $\xi[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(M)] \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi(M)]$.

Conversely assume that $\xi[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(M)] \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi(M)]$, for each TN -set M in R . Let G be a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in Z . Therefore $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G) = G$. By assumption, $\xi[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(\xi^{-1}(G))] \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi(\xi^{-1}(G))] \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G) = G$ and then this implies that $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] \subseteq \xi^{-1}(G)$. Since we also have $\xi^{-1}(G) \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)]$. Thus we conclude that $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] = \xi^{-1}(G)$. This shows that $\xi^{-1}(G)$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in R . Thus, by Theorem 5.3, ξ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping.

Theorem 5.5. A mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping if and only if $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$ for each TN -set G in Z .

Proof. Let $\xi: R \rightarrow Z$ be a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping. Let G be any TN -set set in Z . Since $G \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)$ and therefore we get $\xi^{-1}(G) \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$. So $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] \subseteq \text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G))]$. Since $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)$ is $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in Z and ξ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping. Hence, by Theorem 5.3, $\xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in R . Therefore $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G))] = \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$. Hence, we have $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$.

Conversely, assume $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(G)] \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(G)]$ for all $\text{TN}\alpha^*\text{g}^\#\psi$ -sets G in Z . Let K be a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in Z . By assumption, $\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}[\xi^{-1}(K)] \subseteq \xi^{-1}[\text{TN}\alpha^*\text{g}^\#\psi\text{-Cl}(K)] = \xi^{-1}(K)$. This implies that $\xi^{-1}(K)$ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -closed set in R . Thus by Theorem 5.3, ξ is a $\text{TN}\alpha^*\text{g}^\#\psi$ -irresolute mapping.

Theorem 5.6. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective mapping. Then ξ is $TN\alpha^*g^\# \psi$ -irresolute if and only if $TN\alpha^*g^\# \psi - Int[\xi(M)] \subseteq \xi[TN\alpha^*g^\# \psi - Int(M)]$ for every TN -set M in R .

Proof. Let M be any TN -set in R and ξ be a bijective and $TN\alpha^*g^\# \psi$ -irresolute mapping. Let $\xi(M) = K$. Since $TN\alpha^*g^\# \psi - Int(K) \subseteq K$ and hence we obtain $\xi^{-1}[TN\alpha^*g^\# \psi - Int(K)] \subseteq \xi^{-1}(K)$. Since ξ is an injective mapping, so $\xi^{-1}(K) = M$. Thus $\xi^{-1}[TN\alpha^*g^\# \psi - Int(K)] \subseteq M$. Then this implies that $TN\alpha^*g^\# \psi - Int[\xi^{-1}(TN\alpha^*g^\# \psi - Int(K))] \subseteq TN\alpha^*g^\# \psi - Int(K)$. Since ξ is $TN\alpha^*g^\# \psi$ -irresolute, $\xi^{-1}[TN\alpha^*g^\# \psi - Int(K)]$ is $TN\alpha^*g^\# \psi$ -open set in R and thus $\xi^{-1}[TN\alpha^*g^\# \psi - Int(K)] \subseteq TN\alpha^*g^\# \psi - Int(M)$. Hence $\xi[\xi^{-1}(TN\alpha^*g^\# \psi - Int(K))] \subseteq \xi[TN\alpha^*g^\# \psi - Int(M)]$. Thus $TN\alpha^*g^\# \psi - Int[\xi(M)] \subseteq \xi[TN\alpha^*g^\# \psi - Int(M)]$.

Conversely, suppose that $TN\alpha^*g^\# \psi - Int[\xi(M)] \subseteq \xi[TN\alpha^*g^\# \psi - Int(M)]$ for every TN -set M in R . Let K be a $TN\alpha^*g^\# \psi$ -open set in Z . Since ξ is surjective and therefore, by hypothesis, $K = TN\alpha^*g^\# \psi - Int(K) = TN\alpha^*g^\# \psi - Int[\xi(\xi^{-1}(K))] \subseteq \xi[TN\alpha^*g^\# \psi - Int(\xi^{-1}(K))]$. It follows that $\xi^{-1}(K) \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(K)]$. Therefore $\xi^{-1}(K)$ is $TN\alpha^*g^\# \psi$ -open set in R . Hence ξ is a $TN\alpha^*g^\# \psi$ -irresolute mapping.

Theorem 5.7. A mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is a $TN\alpha^*g^\# \psi$ -irresolute mapping if and only if $\xi^{-1}[TN\alpha^*g^\# \psi - Int(H)] \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(H)]$ for every TN -set H in Z .

Proof. Let H be any TN -set in Z and ξ be a $TN\alpha^*g^\# \psi$ -irresolute mapping. Then $TN\alpha^*g^\# \psi - Int(H) \subseteq H$ implies that $\xi^{-1}[TN\alpha^*g^\# \psi - Int(H)] \subseteq \xi^{-1}(H)$. Thus $TN\alpha^*g^\# \psi - Int[\xi^{-1}(TN\alpha^*g^\# \psi - Int(H))] \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(H)]$. Since $TN\alpha^*g^\# \psi - Int(H)$ is $TN\alpha^*g^\# \psi$ -open set in Z and ξ is $TN\alpha^*g^\# \psi$ -irresolute, $\xi^{-1}[TN\alpha^*g^\# \psi - Int(H)]$ is $TN\alpha^*g^\# \psi$ -open set in R . Thus $TN\alpha^*g^\# \psi - Int[\xi^{-1}(TN\alpha^*g^\# \psi - Int(H))] = \xi^{-1}[TN\alpha^*g^\# \psi - Int(H)]$. Then we obtain $\xi^{-1}[TN\alpha^*g^\# \psi - Int(H)] \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(H)]$.

Conversely, assume that $\xi^{-1}[TN\alpha^*g^\# \psi - Int(H)] \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(H)]$ for every TN -set H in Z . Let M be any $TN\alpha^*g^\# \psi$ -open set in Z . Then by assumption, $\xi^{-1}(M) = \xi^{-1}[TN\alpha^*g^\# \psi - Int(M)] \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(M)]$ and therefore $\xi^{-1}(M) \subseteq TN\alpha^*g^\# \psi - Int[\xi^{-1}(M)]$. This implies that $\xi^{-1}(M)$ is $TN\alpha^*g^\# \psi$ -open set in R . Hence ξ is a $TN\alpha^*g^\# \psi$ -irresolute mapping.

Theorem 5.8. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective mapping and $TN\alpha^*g^\# \psi$ -irresolute mapping. Then $\xi[TN\alpha^*g^\# \psi - Fr(M)] \subseteq TN\alpha^*g^\# \psi - Fr[\xi(M)]$ for every TN -set M in R .

Proof. Let ξ be a bijective and $TN\alpha^*g^\# \psi$ -irresolute mapping. Let M be a TN -set in R . By definition, $TN\alpha^*g^\# \psi - Fr(M) = TN\alpha^*g^\# \psi - Cl(M) \cap TN\alpha^*g^\# \psi - Cl(M^c)$. By Theorem 5.6, $TN\alpha^*g^\# \psi - Int[\xi(M)] \subseteq \xi[TN\alpha^*g^\# \psi - Int(M)]$ and from Theorem 5.4, $\xi[TN\alpha^*g^\# \psi - Cl(M)] \subseteq TN\alpha^*g^\# \psi - Cl[\xi(M)]$. Thus $\xi[TN\alpha^*g^\# \psi - Fr(M)] = \xi[TN\alpha^*g^\# \psi - Cl(M)] \cap \xi[TN\alpha^*g^\# \psi - Cl(M^c)] \subseteq TN\alpha^*g^\# \psi - Cl[\xi(M)] \cap TN\alpha^*g^\# \psi - Cl[\xi(M)^c] = TN\alpha^*g^\# \psi - Fr[\xi(M)]$.

Theorem 5.9. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective $TN\alpha^*g^\# \psi$ -irresolute mapping. Then we have $TN\alpha^*g^\# \psi - Fr[\xi^{-1}(M)] \subseteq \xi^{-1}[TN\alpha^*g^\# \psi - Fr(M)]$ for every TN -set M in Z .

Proof. Let ξ be a bijective and $TN\alpha^*g^\# \psi$ -irresolute mapping. Let M be a TN -set in Z . By Theorem 5.5, $TN\alpha^*g^\# \psi - Cl[\xi^{-1}(M)] \subseteq \xi^{-1}[TN\alpha^*g^\# \psi - Cl(M)]$. Now we have $\xi^{-1}[TN\alpha^*g^\# \psi - Fr(M)] = \xi^{-1}[(TN\alpha^*g^\# \psi - Cl(M)) \cap TN\alpha^*g^\# \psi - Cl(M^c)] = \xi^{-1}[TN\alpha^*g^\# \psi - Cl(M)] \cap \xi^{-1}[TN\alpha^*g^\# \psi - Cl(M^c)] \supseteq TN\alpha^*g^\# \psi - Cl[\xi^{-1}(M)] \cap TN\alpha^*g^\# \psi - Cl[\xi^{-1}(M^c)] = TN\alpha^*g^\# \psi - Cl[\xi^{-1}(M)] \cap TN\alpha^*g^\# \psi - Cl[(\xi^{-1}(M))^c] = TN\alpha^*g^\# \psi - Fr[\xi^{-1}(M)]$. Therefore, we obtain $TN\alpha^*g^\# \psi - Fr[\xi^{-1}(M)] \subseteq \xi^{-1}[TN\alpha^*g^\# \psi - Fr(M)]$.

Theorem 5.10. If $\xi: (R, T_{TN}) \rightarrow (W, \Gamma_{TN})$ and $\mu: (W, \Gamma_{TN}) \rightarrow (Z, \Psi_{TN})$ are $TN\alpha^*g^\# \psi$ -irresolute, then the composition function $\mu \circ \xi: (R, T_{TN}) \rightarrow (Z, \Psi_{TN})$ is also $TN\alpha^*g^\# \psi$ -irresolute.

Proof. Let M be a $TN\alpha^*g^\# \psi$ -open set in Z . Since μ is a $TN\alpha^*g^\# \psi$ -irresolute mapping, $\mu^{-1}(M)$ is $TN\alpha^*g^\# \psi$ -open in W . Since ξ is a $TN\alpha^*g^\# \psi$ -irresolute mapping, $\xi^{-1}[\mu^{-1}(M)] = (\mu \circ \xi)^{-1}(M)$ is $TN\alpha^*g^\# \psi$ -open in R . Therefore $\mu \circ \xi$ is $TN\alpha^*g^\# \psi$ -irresolute.

Theorem 5.11. If $\xi: (R, T_{TN}) \rightarrow (W, \Gamma_{TN})$ and is $TN\alpha^*g^\# \psi$ -irresolute and $\mu: (W, \Gamma_{TN}) \rightarrow (Z, \Psi_{TN})$ is $TN\alpha^*g^\# \psi$ -continuous, then the composition function $\mu \circ \xi: (R, T_{TN}) \rightarrow (Z, \Psi_{TN})$ is also a $TN\alpha^*g^\# \psi$ -continuous function.

Proof. Let M be a TN -open set in Z . Since μ is a $TN\alpha^*g^\# \psi$ -continuous mapping, $\mu^{-1}(M)$ is $TN\alpha^*g^\# \psi$ -open set in W . Since ξ is a $TN\alpha^*g^\# \psi$ -irresolute function, $\xi^{-1}[\mu^{-1}(M)] = (\mu \circ \xi)^{-1}(M)$ is $TN\alpha^*g^\# \psi$ -open in R . Therefore $\mu \circ \xi$ is $TN\alpha^*g^\# \psi$ -continuous.

6. Tetra Neutrosophic $\alpha^*g^\# \psi$ -Closed and Open Mappings

In this section, we introduce Tetra Neutrosophic $\alpha^*g^\# \psi$ -closed (briefly $TN\alpha^*g^\# \psi$ -closed) functions and Tetra Neutrosophic $\alpha^*g^\# \psi$ -open (briefly $TN\alpha^*g^\# \psi$ -open) mappings in TN -Top-Spaces and investigate certain characterizations and properties of these classes of mappings.

Definition 6.1. A mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is said to be $TN\alpha^*g^\# \psi$ -closed if the image of each TN -closed set in R is $TN\alpha^*g^\# \psi$ -closed in Z .

Definition 6.2. A function $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is said to be $TN\alpha^*g^\# \psi$ -open if the image of each TN -open set in R is $TN\alpha^*g^\# \psi$ -open in Z .

Theorem 6.3. A function $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -closed if and only if $TN\alpha^*g^\# \psi - Cl[\xi(A)] \subseteq \xi[TN - Cl(A)]$ for every TN -set M of R .

Proof. Suppose $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is a $TN\alpha^*g^\# \psi$ -closed function and M is any TN -set in R . Then $TN - Cl(M)$ is a TN -closed set in R . Since ξ is $TN\alpha^*g^\# \psi$ -closed, $\xi[TN - Cl(M)]$ is a $TN\alpha^*g^\# \psi$ -closed

set in Z . So we have $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(\text{TN}-\text{Cl}(M))] = \xi[\text{TN}-\text{Cl}(M)]$. Therefore, $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(M)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(\text{TN}-\text{Cl}(M))] = \xi(\text{TN}-\text{Cl}(M))$. Thus we obtain $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(M)] \subseteq \xi(\text{TN}-\text{Cl}(M))$. Conversely, let M be a TN -closed set in R . Then $\text{TN}-\text{Cl}(M) = M$ and so $\xi(M) = \xi[\text{TN}-\text{Cl}(M)]$. By our assumption $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(M)] \subseteq \xi(M)$. But $\xi(M) \subseteq \text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(M)]$. Thus we conclude that $\text{TN}\alpha^*g^\#\psi - \text{Cl}[\xi(M)] = \xi(M)$ and hence $\xi(M)$ is $\text{TN}\alpha^*g^\#\psi$ -closed set in Z . Therefore eventually ξ is a $\text{TN}\alpha^*g^\#\psi$ -closed mapping.

Theorem 6.4. A mapping $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is $\text{TN}\alpha^*g^\#\psi$ -closed if and only if for each TN -set H of Z and for each TN -open set G of R containing $\xi^{-1}(H)$ there exists a $\text{TN}\alpha^*g^\#\psi$ -open set V of Z such that $H \subseteq V$ and $\xi^{-1}(V) \subseteq G$.

Proof. Let ξ be a $\text{TN}\alpha^*g^\#\psi$ -closed mapping and let H be any TN -set in Z and G be a TN -open set of R such that $\xi^{-1}(H) \subseteq G$. Then $V = [\xi(G^c)]^c$ is $\text{TN}\alpha^*g^\#\psi$ -open in Z such that $\xi^{-1}(V) \subseteq G$.

Conversely, let H be a TN -closed of R . Then $\xi^{-1}[(\xi(H))^c] \subseteq H^c$ and H^c is TN -open in R . By assumption, there exists a $\text{TN}\alpha^*g^\#\psi$ -open set V of Z such that $[\xi(H)]^c \subseteq V$ and $\xi^{-1}(V) \subseteq H^c$ and this implies that $H \subseteq [\xi^{-1}(V)]^c$. Hence we obtain $V^c \subseteq \xi(H) \subseteq \xi[(\xi^{-1}(V))^c] \subseteq V^c$, which implies $\xi(H) = V^c$. Since V^c is $\text{TN}\alpha^*g^\#\psi$ -closed, $\xi(H)$ is $\text{TN}\alpha^*g^\#\psi$ -closed and ξ is $\text{TN}\alpha^*g^\#\psi$ -closed mapping.

Theorem 6.5. Let $\xi: (R, T_{\text{TN}}) \rightarrow (W, \Gamma_{\text{TN}})$ be a TN -closed mapping and $\mu: (W, \Gamma_{\text{TN}}) \rightarrow (Z, \Psi_{\text{TN}})$ be a $\text{TN}\alpha^*g^\#\psi$ -closed mapping. Then their composition $\mu \circ \xi: (R, T_{\text{TN}}) \rightarrow (Z, \Psi_{\text{TN}})$ is a $\text{TN}\alpha^*g^\#\psi$ -closed mapping.

Proof. Let M be a TN -closed set in R . Since ξ is TN -closed, $\xi(M)$ is TN -closed in W . Since μ is $\text{TN}\alpha^*g^\#\psi$ -closed, $\mu[\xi(M)] = (\mu \circ \xi)(M)$ is $\text{TN}\alpha^*g^\#\psi$ -closed in Z . Hence $\mu \circ \xi$ is a $\text{TN}\alpha^*g^\#\psi$ -closed mapping.

Theorem 6.6. Let $\xi: (R, T_{\text{TN}}) \rightarrow (W, \Gamma_{\text{TN}})$ and $\mu: (W, \Gamma_{\text{TN}}) \rightarrow (Z, \Psi_{\text{TN}})$ be two mappings such that their composition $\mu \circ \xi: (R, T_{\text{TN}}) \rightarrow (Z, \Psi_{\text{TN}})$ is $\text{TN}\alpha^*g^\#\psi$ -closed. Then the following statements are true.

- (i) If ξ is TN -continuous and surjective, then μ is $\text{TN}\alpha^*g^\#\psi$ -closed.
- (ii) If μ is $\text{TN}\alpha^*g^\#\psi$ -irresolute and injective, then ξ is $\text{TN}\alpha^*g^\#\psi$ -closed.

Proof. (i) Let M be a TN -closed set of W . Since ξ is TN -continuous, $\xi^{-1}(M)$ is TN -closed in R . Since $\mu \circ \xi$ is $\text{TN}\alpha^*g^\#\psi$ -closed, $(\mu \circ \xi)(\xi^{-1}(M))$ is $\text{TN}\alpha^*g^\#\psi$ -closed in Z . Since ξ is surjective, $(\mu \circ \xi)(\xi^{-1}(M)) = \mu(M)$ is $\text{TN}\alpha^*g^\#\psi$ -closed in Z . Hence μ is $\text{TN}\alpha^*g^\#\psi$ -closed.

(ii) Let M be any TN -closed set of R . Since $\mu \circ \xi$ is $\text{TN}\alpha^*g^\#\psi$ -closed, $(\mu \circ \xi)(M)$ is $\text{TN}\alpha^*g^\#\psi$ -closed in Z . Since μ is $\text{TN}\alpha^*g^\#\psi$ -irresolute, $\mu^{-1}(\mu \circ \xi(M))$ is $\text{TN}\alpha^*g^\#\psi$ -closed in W . Since μ is injective, $\mu^{-1}(\mu \circ \xi(M)) = \mu^{-1}(\mu(\xi(M))) = \xi(M)$ is $\text{TN}\alpha^*g^\#\psi$ -closed in W . Hence ξ is $\text{TN}\alpha^*g^\#\psi$ -closed.

Theorem 6.7. A function $\xi: (R, T_{\text{TN}}) \rightarrow (Z, \Gamma_{\text{TN}})$ is $\text{TN}\alpha^*g^\#\psi$ -open if and only if $\xi[\text{TN}-\text{Int}(M)] \subseteq \text{TN}\alpha^*g^\#\psi - \text{Int}[\xi(M)]$, for every TN -set M of R .

Proof. Suppose $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is a $TN\alpha^*g^\# \psi$ -open function and M is any TN -set in R . Then $TN-Int(M)$ is a TN -open set in R . Since ξ is $TN\alpha^*g^\# \psi$ -open, $\xi[TN-Int(M)]$ is a $TN\alpha^*g^\# \psi$ -open set. Since $TN\alpha^*g^\# \psi - Int[\xi(TN-Int(M))] \subseteq TN\alpha^*g^\# \psi - Int[\xi(M)]$, $\xi[TN-Int(M)] \subseteq TN\alpha^*g^\# \psi - Int[\xi(M)]$. Conversely, $\xi[TN-Int(M)] \subseteq TN\alpha^*g^\# \psi - Int[\xi(M)]$ for every TN -set M in R . Let E be a TN -open set in R . Then $TN-Int(E) = E$ and by hypothesis, $\xi(E) \subseteq TN\alpha^*g^\# \psi - Int[\xi(E)]$. But $TN\alpha^*g^\# \psi - Int[\xi(E)] \subseteq \xi(E)$. Therefore, $\xi(E) = TN\alpha^*g^\# \psi - Int[\xi(E)]$. Then $\xi(E)$ is $TN\alpha^*g^\# \psi$ -open. Hence ξ is a $TN\alpha^*g^\# \psi$ -open mapping.

Theorem 6.8. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha^*g^\# \psi$ -open function. If for each $p_{(t,i,f)} \in TN(R)$ and for each TN -neighbourhood M of $p_{(t,i,f)}$ in R , there exists a $TN\alpha^*g^\# \psi$ -neighbourhood G of $\xi(p_{(t,i,f)})$ in Z such that $G \subseteq \xi(M)$.

Proof. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha^*g^\# \psi$ -open function. Let $p_{(t,i,f)} \in TN(R)$ and M be any arbitrary TN -neighbourhood of $p_{(t,i,f)}$ in R . Then there exists a TN -open set G such that $p_{(t,i,f)} \in G \subseteq M$. By Theorem 6.7, $\xi(G) = \xi[TN-Int(G)] \subseteq TN\alpha^*g^\# \psi - Int[\xi(G)]$. But, $TN\alpha^*g^\# \psi - Int[\xi(G)] \subseteq \xi(G)$. Therefore, $TN\alpha^*g^\# \psi - Int[\xi(G)] = \xi(G)$ and hence $\xi(G)$ is $TN\alpha^*g^\# \psi$ -open in Z . Since $p_{(t,i,f)} \in G \subseteq M$, $\xi(p_{(t,i,f)}) \in \xi(G) \subseteq \xi(M)$ and so the result follows by taking $H = \xi(G)$.

Theorem 6.9. For any bijective mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ the following statements are equivalent:

- (i) $\xi^{-1}: (Z, \Gamma_{TN}) \rightarrow (R, T_{TN})$ is $TN\alpha^*g^\# \psi$ -continuous.
- (ii) ξ is $TN\alpha^*g^\# \psi$ -open.
- (iii) ξ is $TN\alpha^*g^\# \psi$ -closed.

Proof. (i) $\Rightarrow$ (ii): Let M be a TN -open set in R . By assumption, $(\xi^{-1})^{-1}(M) = \xi(M)$ is $TN\alpha^*g^\# \psi$ -open in Z and hence ξ is a $TN\alpha^*g^\# \psi$ -open function.

(ii) $\Rightarrow$ (iii): Let M be a TN -closed set of R . Then M^c is a TN -open set in R . By assumption $\xi(M^c)$ is $TN\alpha^*g^\# \psi$ -open in Z . But $\xi(M^c) = [\xi(M)]^c$. Therefore $\xi(M)$ is $TN\alpha^*g^\# \psi$ -closed set in Z . Hence, ξ is $TN\alpha^*g^\# \psi$ -closed.

(iii) $\Rightarrow$ (i): Let M be a TN -closed set of R . By assumption, $\xi(M)$ is $TN\alpha^*g^\# \psi$ -closed set in Z . But $\xi(M) = (\xi^{-1})^{-1}(M)$ and therefore by Theorem 4.4, $\xi^{-1}: Z \rightarrow R$ is $TN\alpha^*g^\# \psi$ -continuous.

7. Tetra Neutrosophic $TN\alpha^*g^\# \psi$ -Homeomorphisms

In this section, we define and study the concept of Tetra Neutrosophic $\alpha^*g^\# \psi$ -homeomorphism (briefly, $TN\alpha^*g^\# \psi$ -homeomorphism) in TN -Top-Space (R, T_{TN}) and obtain some of its properties and characterizations.

Definition 7.1. A mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is said to be TN -homeomorphism if ξ is one-to-one & onto, TN -open and TN -continuous.

Definition 7.2. A mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is said to be $TN\alpha^*g^\# \psi$ -homeomorphism if ξ is one-to-one and onto, $TN\alpha^*g^\# \psi$ -open and $TN\alpha^*g^\# \psi$ -continuous.

Theorem 7.3. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a one-to-one onto mapping. Then ξ is $TN\alpha^*g^\# \psi$ -homeomorphism if and only if ξ is $TN\alpha^*g^\# \psi$ -closed and $TN\alpha^*g^\# \psi$ -continuous.

Proof. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha^*g^\# \psi$ -homeomorphism. Then ξ is $TN\alpha^*g^\# \psi$ -continuous. Let M be an arbitrary TN -closed set in (R, T_{TN}) . Then $1_{TN} - M$ is TN -open. Since ξ is $TN\alpha^*g^\# \psi$ -open, $\xi(1_{TN} - M)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) . That is, $1_{TN} - \xi(M)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) for every TN -closed set M in (R, T_{TN}) implies that $\xi(M)$ is $TN\alpha^*g^\# \psi$ -closed in (Z, Γ_{TN}) . Hence $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -closed.

Conversely, let ξ be a $TN\alpha^*g^\# \psi$ -closed and $TN\alpha^*g^\# \psi$ -continuous function. Let M be a TN -open set in (R, T_{TN}) . Then $1_{TN} - M$ is TN -closed in (R, T_{TN}) . Since ξ is $TN\alpha^*g^\# \psi$ -closed, $\xi(1_{TN} - M)$ is $TN\alpha^*g^\# \psi$ -closed in (Z, Γ_{TN}) . That is, $\xi(1_{TN} - M) = 1_{TN} - \xi(M)$ is $TN\alpha^*g^\# \psi$ -closed in (Z, Γ_{TN}) . Hence $\xi(M)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) for every TN -open set M in (R, T_{TN}) . Thus, ξ is $TN\alpha^*g^\# \psi$ -open and hence $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -homeomorphism.

Theorem 7.4.: Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective and $TN\alpha^*g^\# \psi$ -continuous mapping. Then the following statements are equivalent.

- (i) ξ is a $TN\alpha^*g^\# \psi$ -open mapping.
- (ii) ξ is a $TN\alpha^*g^\# \psi$ -homeomorphism.
- (iii) ξ is a $TN\alpha^*g^\# \psi$ -closed mapping.

Proof. (i) $\Rightarrow$ (ii). By the given hypothesis, the mapping $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is bijective $TN\alpha^*g^\# \psi$ -continuous and $TN\alpha^*g^\# \psi$ -open. Hence $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -homeomorphism.

(ii) $\Rightarrow$ (iii). Let M be a TN -closed set in (R, T_{TN}) . Then M^c is FQN -open in (R, T_{TN}) . By assumption, $\xi(M^c)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) . That is $\xi(M^c) = (\xi(M))^c = Z - \xi(M)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) and hence $\xi(M)$ is $TN\alpha^*g^\# \psi$ -closed in (Z, Γ_{TN}) for every TN -closed set M in (R, T_{TN}) . Hence the function $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -closed mapping.

(iii) $\Rightarrow$ (i). Let M be a TN -open set in (R, T_{TN}) . Then M^c is TN -closed set in (R, T_{TN}) . By the given hypothesis, $\xi(M^c)$ is $TN\alpha^*g^\# \psi$ -closed in (Z, Γ_{TN}) . But $\xi(M^c) = (\xi(M))^c = 1_{TN} - \xi(M)$ is $TN\alpha^*g^\# \psi$ -closed. Therefore $\xi(M)$ is $TN\alpha^*g^\# \psi$ -open in (Z, Γ_{TN}) for every TN -open set M in (R, T_{TN}) . Hence $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -open mapping.

Theorem 7.5. Every TN -homeomorphism is $TN\alpha^*g^\# \psi$ -homeomorphism.

Proof. If $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is TN -homeomorphism, by definition 7.1, ξ is bijective, TN -continuous and TN -open. Then $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -continuous and $TN\alpha^*g^\# \psi$ -open. Hence the function $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\# \psi$ -homeomorphism. Every TN -continuous function is $TN\alpha^*g^\# \psi$ -continuous and every TN -open map is $TN\alpha^*g^\# \psi$ -open. Then ξ is bijective, $TN\alpha^*g^\# \psi$ -continuous and $TN\alpha^*g^\# \psi$ -open. Therefore ξ is $TN\alpha^*g^\# \psi$ -homeomorphism.

Theorem 7.6. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha$ -homeomorphism. Then prove that ξ is $TN\alpha^*g^\#\psi$ -homeomorphism.

Proof. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a $TN\alpha$ -homeomorphism. Then ξ is bijective, $TN\alpha$ -continuous and $TN\alpha$ -open. Let H be a TN -open set in (Z, Γ_{TN}) . Since ξ is $TN\alpha$ -continuous, $\xi^{-1}(H)$ is $TN\alpha$ -open set in (R, T_{TN}) . Since every $TN\alpha$ -open set is $TN\alpha^*g^\#\psi$ -open set, $\xi^{-1}(H)$ is $TN\alpha^*g^\#\psi$ -open in (R, T_{TN}) which implies that ξ is $TN\alpha^*g^\#\psi$ -continuous. Let M be a TN -open set in (R, T_{TN}) . Since ξ is $TN\alpha$ -open, $\xi(M)$ is $TN\alpha$ -open set in (Z, Γ_{TN}) . Since every $TN\alpha$ -open set is $TN\alpha^*g^\#\psi$ -open set, $\xi(M)$ is $TN\alpha^*g^\#\psi$ -open set in (Z, Γ_{TN}) which implies ξ is $TN\alpha^*g^\#\psi$ -open mapping. Therefore, consequently ξ is $TN\alpha^*g^\#\psi$ -homeomorphism.

Theorem 7.7. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective and $TN\alpha^*g^\#\psi$ -irresolute mapping. Then the following statements are equivalent.

- (i) ξ is a $TN\alpha^*g^\#\psi$ -open mapping.
- (ii) ξ is a $TN\alpha^*g^\#\psi$ -homeomorphism.
- (iii) ξ is a $TN\alpha^*g^\#\psi$ -closed mapping.

Proof. Identical to the proof of Theorem 7.4.

Theorem 7.8. Let $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ be a bijective function. Then prove that ξ is $TN\alpha^*g^\#\psi$ -homeomorphism if and only if $\xi[TN\alpha^*g^\#\psi - Cl(M)] = TN - Cl(\xi(M))$ for every subset M of (R, T_{TN}) .

Proof. If $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\#\psi$ -homeomorphisms, then ξ is $TN\alpha^*g^\#\psi$ -continuous and $TN\alpha^*g^\#\psi$ -closed. If $M \subseteq R$. Then $\xi[TN\alpha^*g^\#\psi - Cl(M)] \subseteq TN - Cl(\xi(M))$ since ξ is $TN\alpha^*g^\#\psi$ -continuous. Since $TN\alpha^*g^\#\psi - Cl(A)$ is TN -closed in (R, T_{TN}) and ξ is $TN\alpha^*g^\#\psi$ -closed mapping, $\xi[TN\alpha^*g^\#\psi - Cl(M)]$ is $TN\alpha^*g^\#\psi$ -closed in (Z, Γ_{TN}) . Also $TN\alpha^*g^\#\psi - Cl[\xi(TN\alpha^*g^\#\psi - Cl(M))] = \xi(TN\alpha^*g^\#\psi - Cl(M))$. Since $M \subseteq TN\alpha^*g^\#\psi - Cl(M)$, $\xi(M) \subseteq \xi[TN\alpha^*g^\#\psi - Cl(M)]$ and therefore we obtain $TN - Cl(\xi(M)) \subseteq TN - Cl[\xi(TN\alpha^*g^\#\psi - Cl(M))] = \xi(TN\alpha^*g^\#\psi - Cl(M))$. Thus $TN - Cl(\xi(M)) \subseteq \xi(TN\alpha^*g^\#\psi - Cl(M))$. Hence, $\xi[TN\alpha^*g^\#\psi - Cl(M)] = TN - Cl(\xi(M))$ if ξ is $TN\alpha^*g^\#\psi$ -homeomorphism.

Conversely, if $\xi[TN\alpha^*g^\#\psi - Cl(M)] = TN - Cl(\xi(M))$ for every subset M of (R, T_{TN}) , then ξ is $TN\alpha^*g^\#\psi$ -continuous. If M is TN -closed in (R, T_{TN}) , then M is $TN\alpha^*g^\#\psi$ -closed in (R, T_{TN}) . Then $TN\alpha^*g^\#\psi - Cl(M) = M$ which implies $\xi[TN\alpha^*g^\#\psi - Cl(M)] = \xi(M)$. Hence, by the given hypothesis, it follows that $TN - Cl(\xi(M)) = \xi(M)$. Thus $\xi(M)$ is TN -closed in Z and hence $TN\alpha^*g^\#\psi$ -closed in Z for every TN -closed set M in R . That is, ξ is a $TN\alpha^*g^\#\psi$ -closed mapping. Therefore $\xi: (R, T_{TN}) \rightarrow (Z, \Gamma_{TN})$ is $TN\alpha^*g^\#\psi$ -homeomorphism.

Conclusion

We introduced and systematically investigated the concept of Tetra Neutrosophic α^* generalized $\mathcal{G}^\# \psi$ -closed sets in Tetra Neutrosophic topological spaces. The fundamental properties of $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -closed sets were established through a collection of results concerning their relationships with the relevant generalized closed and open sets and their associated closure properties. These results provide a systematic theoretical foundation for the subsequent development of generalized mappings in Tetra Neutrosophic topological spaces.

Based on the proposed $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -closed sets, we introduced and investigated $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -continuous, $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -irresolute, $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -open and $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -closed mappings. Their basic properties, relationships, and characterizations were established within the Tetra Neutrosophic topological framework. Furthermore, the concept of $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -homeomorphism was introduced, and fundamental properties of this homeomorphism were investigated, thereby providing a suitable notion of topological equivalence based on the proposed generalized structure.

Overall, the results obtained in this paper extend the theory of α^* generalized $\mathcal{G}^\# \psi$ -closed sets and their associated mappings from classical topological spaces to the Tetra Neutrosophic setting. The study consequently establishes a coherent framework connecting $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -closed sets with generalized continuity, irresoluteness, openness, closedness, and homeomorphisms. Thus, the present work contributes to the development of generalized mapping theory in the newly developing field of Tetra Neutrosophic topology and provides a foundation for the further study of generalized topological structures in this setting.

Recommendations

It is recommended to investigate the relationships of $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -mappings with other generalized mappings and generalized closed and open sets in Tetra Neutrosophic topology. Further research may examine separation axioms, compactness, connectedness, countability properties, and various forms of generalized compactness associated with $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -mappings.

Another promising direction is to study the preservation of these properties under composition and inverse mappings and to characterize $\text{TN}\alpha^* \mathcal{G}^\# \psi$ -homeomorphisms through suitable invariants of Tetra Neutrosophic topological spaces. The proposed framework may also be extended to other neutrosophic structures and, where appropriate, investigated for applications involving uncertain, inconsistent, and indeterminate information in intelligent systems, decision-making, and information processing.

These directions may provide further theoretical development of Tetra Neutrosophic topology and contribute to the establishment of a broader framework for generalized mappings in uncertainty-aware mathematical models.

Scientific Ethics Declaration

* The author declares that the scientific ethical and legal responsibility of this article published in EPSTEM Journal belongs to the author.

Conflict of Interest

* The author declares that there is no conflict of interest.

Funding

* This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Acknowledgements or Notes

* This article was presented as an oral presentation at the International Conference on Basic Sciences, Engineering and Technology (www.icbaset.net) held in Istanbul/Türkiye on September 17-18, 2026.

* The author is indebted to Prince Mohammad Bin Fahd University Al Khobar, Saudi Arabia, for providing the necessary research facilities during the preparation of this research paper.

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To cite this article:

Latif, R. M. (2026). Tetra neutrosophic alpha star generalized $\# \Psi$ mappings in tetra neutrosophic topological spaces *The Eurasia Proceedings of Science, Technology, Engineering and Mathematics (EPSTEM)*, 42, 237-259.